SHAROF RASHIDOV NOMIDAGI
SAMARQAND DAVLAT UNIVERSITETI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.03/2025.27.12.FM/T.09.03 RAQAMLI ILMIY KENGASH

SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

FAYZIYEV BEKZODJON MURTAZAYEVICH

BIRJINSLIMAS SUYUQLIKLARNING G‘OVAK MUHITLARDA SIZISHI JARAYONLARINI KO‘P BOSQICHLI KINETIKA ASOSIDA TADQIQ ETISH

01.02.05 – Suyuqlik va gaz mexanikasi

FIZIKA-MATEMATIKA FANLARI DOKTORI (DSc)
DISSERTATSIYASI AVTOREFERATI

Samarqand – 2026

UDK:532.546

Fan doktori (DSc) dissertatsiyasi avtoreferati mundarijasi

Contents of dissertation abstract of the doctor of science (DSc)

Оглавление автореферата диссертации доктора наук (DSc)

SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.03/2025.27.12.FM/T.09.03 RAQAMLI ILMIY KENGASH

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Fan doktori (DSc) dissertatsiyasi mavzusi O'zbekiston Respublikasi Fanlar Akademiyasi huzuridagi Oliy attestatsiya komissiyasida B2026.1.DSc/FM318 raqam bilan ro'yxatga olingan.

Dissertatsiya Sharof Rashidov nomidagi Samarqand davlat universitetida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o'zbek, ingliz, rus (rezyume)) Ilmiy kengash veb-sahifasida (www.samdu.uz) va "Ziyonet" axborot ta'lim portalida (www.ziyonet.uz) joylashtirilgan.

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Dissertatsiya avtoreferati 2026 yil «26» avgust kuni tarqatildi.
(2026 yil «26» avgust dagi 1 raqamli reyestr bayonnomasi).

O'ZBEKISTON RESPUBLIKASI OLIY TA'LIM, FAN VA INNOVATSIYALAR VAZIRLIGI • SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI • O'ZBEKISTON

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Dissertatsiya mavzusining dolzarbligi va zarurati. Jahonda suv resurslaridan oqilona foydalanish, yer osti suvlarini muhofaza qilish va g‘ovak muhitlarda modda ko‘chishi jarayonlarini chuqur o‘rganishga alohida e’tibor berilmoqda. Birlashgan Millatlar Tashkilotining ma’lumotlariga ko‘ra, “2024-yil holatida dunyoda 2,2 milliard kishi xavfsiz ichimlik suvi xizmatlaridan foydalanish imkoniyatiga ega emas”[1]. Suv resurslarining sifat jihatidan yomonlashuvi, g‘ovak muhitlarda ifloslantiruvchi moddalarning tarqalishi, yer osti suvlarining minerallashuvi va antropogen ta’sirlarning kuchayishi modda ko‘chishi jarayonlarini aniq matematik modellashtirish zaruratini yuzaga keltirmoqda. Bu borada, birjinslimas suyuqliklarning g‘ovak muhitlarda harakatlanishi modellarini ko‘plab fizik, mexanik va kimyoviy omillarni hisobga olgan holda takomillashtirishga alohida e’tibor qaratilmoqda.

Jahonda neft-gaz konlarining samarali o‘zlashtirilishini ta’minlash, yer osti suvlarining harakatini prognozlash, kimyoviy reagentlarning g‘ovak muhitlarda tarqalishini baholash va ekologik xavflarni kamaytirishga qaratilgan ilmiy tadqiqotlar olib borilmoqda. Ushbu yo‘nalishda, jumladan, ko‘plab texnologik va ishlab chiqarish jarayonlarida g‘ovak va yoriq-g‘ovak muhitlarda bir jinsli bo‘lmagan suyuqliklarning sizish muammolarining vujudga kelishi sababli, dispers tizimlarning g‘ovak muhitlarda sizishi jarayonlarini modellashtirish bo‘yicha tadqiqotlar ustuvor hisoblanmoqda. Shu bilan birga, suv resurslariga bo‘lgan talabning global ravishda oshib borayotganligi, neft va gaz qazib olish jarayonida suv haydash bilan bog‘liq ikkilamchi usulning ko‘proq ishlatilayotganligi sababli g‘ovak muhitlarda bir jinsli bo‘lmagan suyuqliklar sizishi va modda ko‘chishi jarayonlarining matematik modellarini takomillashtirish, masalalarni yechishning samarali algoritmlarini yaratish dolzarb vazifalardan hisoblanmoqda.

Respublikamizda g‘ovak muhitlarda modda ko‘chishi jarayonlarini o‘rganishga qaratilgan fundamental va amaliy tadqiqotlar doirasida zamonaviy matematik modellashtirish yondashuvlarini joriy etish, hisoblash tajribalarini o‘tkazish hamda natijalarni vizual tahlil qilishga asoslangan avtomatlashtirilgan dasturiy vositalarni yaratish bo‘yicha izlanishlar olib borilmoqda. Respublikamizda “… ichimlik suvi ta’minoti va oqava suv xizmatlari sohasi faoliyatini boshqarishning zamonaviy usullarini joriy etishni ilmiy-uslubiy qo‘llab-quvvatlash hamda sohaga innovatsion texnologik yechimlar, ilmiy asoslangan zamonaviy yondashuvlarni joriy etish” ga alohida e’tibor qaratilmoqda. Jumladan, O‘zbekiston Respublikasi Prezidentining «O‘zbekiston Respublikasi suv xo‘jaligini rivojlantirishning 2020–2030-yillarga mo‘ljallangan konsepsiyasini tasdiqlash to‘g‘risida[2]» gi 2020-yil 10-iyuldagi PF-6024-sonli Farmon hamda «O‘zbekiston Respublikasida ichimlik suvi ta’minoti va kanalizatsiya tizimlarini rivojlantirish

[1] https://www.un.org/sustainabledevelopment/water-and-sanitation/

[2] O‘zbekiston Respublikasi Prezidentining 2020 yil 10 iyuldagi PF-6024-son “O‘zbekiston Respublikasi suv xo‘jaligini rivojlantirishning 2020 - 2030-yillarga mo‘ljallangan konsepsiyasini tasdiqlash to‘g‘risida”gi Farmoni.

bo‘yicha qo‘shimcha chora-tadbirlar to‘g‘risida[3]» gi 2019-yil 30-oktabrdagi PQ-4040-sonli Qarorda suv resurslarini boshqarishda ilmiy asoslangan yondashuvlarni joriy etish, zamonaviy texnologiyalar va matematik modellashtirish usullaridan foydalanish muhim vazifa sifatida belgilangan. Ushbu vazifalarni amalga oshirishda, xususan, tejamkor va ekologik xavfsiz materiallar asosida yangi suv tozalash quvvatlarini qurish va mavjudlarini modernizatsiya qilish, jumladan, suv tozalash inshootlarida ishlatiladigan filtrlarning optimal parametrlarini matematik modellashtirish va zamonaviy axborot texnologiyalarini qo‘llagan holda aniqlash muhim hisoblanadi.

O‘zbekiston Respublikasi Prezidentining 2022-yil 28-yanvardagi PF-60-son «2022–2026 yillarga mo‘ljallangan Yangi O‘zbekistonning taraqqiyot strategiyasi to‘g‘risida», 2020-yil 10-iyuldagi PF-6024-son «O‘zbekiston Respublikasi suv xo‘jaligini rivojlantirishning 2020–2030-yillarga mo‘ljallangan konsepsiyasini tasdiqlash to‘g‘risida»gi Farmonlari, 2023-yil 24-oktabrdagi PQ-343-son “Ichimlik suv ta’minoti va oqova suv tizimini yanada takomillashtirish bo‘yicha qo‘shimcha chora-tadbirlar to‘g‘risida”gi Qarorlari hamda mazkur faoliyatga tegishli boshqa me’yoriy-huquqiy hujjatlarda nazarda tutilgan maqsad va vazifalarni amalga oshirishga ushbu dissertatsiya ishi muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishiga mosligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. «Matematika, mexanika va informatika» ustuvor yo‘nalishi doirasida bajarilgan.

Dissertatsiya mavzusi bo‘yicha xorijiy ilmiy tadqiqotlar sharhi.[4]

Bir jinsli bo‘lmagan suyuqliklarning g‘ovak muhitlarda sizishi jarayonlari bo‘yicha tadqiqotlar dunyoning yetakchi ilmiy markazlari va oliy ta’lim muassasalarida, jumladan, Amerika neft instituti (API), Texas universiteti, Adelaida universiteti, Yangi Janubiy Uils universiteti (AQSH), Neft muhandisligi instituti (Germaniya), Norvegiya fan va texnika universiteti (Norvegiya), Neft va gaz instituti (Shotlandiya), Delft texnologiya universiteti (Niderlandiya), Oksford energetik tadqiqotlar instituti (Angliya), Xitoy neft universiteti (Xitoy), Yangi Janubiy Uils universiteti (Avstraliya), PETRONAS texnologik universiteti (Malaziya), Meksika neft instituti (Meksika), Qirol Fahd nomidagi neft va foydali qazilmalar universiteti (Saudiya Arabistoni), RFA SB neft kimyosi instituti, Akademik A. P. Krilov nomidagi Butunrossiya nefte-gaz ilmiy-tadqiqot instituti, I. M. Gubkin nomidagi Rossiya Davlat neft va gaz universiteti, Neft-gaz ilmiy-tadqiqot loyihalari (Ozarbayjon); Neft sanoati ilmiy-tadqiqot instituti, Toshkent Davlat texnika universiteti, Samarqand Davlat universiteti, Qarshi Davlat universiteti, Gubkin nomidagi Rossiya neft-gaz instituti Toshent filiali (O‘zbekiston).

[3] O‘zbekiston Respublikasi Prezidentining 2018 yil 30 noyabrdagi PQ-4040-son “O‘zbekiston Respublikasida ichimlik suvi ta’minoti va kanalizatsiya tizimlarini rivojlantirish bo‘yicha qo‘shimcha chora-tadbirlar to‘g‘risida” gi Qarori.

[4] Dissertatsiya mavzusi bo‘yicha xorijiy ilmiy-tadqiqotlar sharhi, http://www.mathnet.ru; http://msp.org/jomms/about/cover/cover.html; http://link.springer.com; http://www.sciencedirect.com; http://www.dissercat.com/catalog/fiziko-matematicheskie-nauki va boshqa manbalar asosida ishlab chiqilgan.

Dunyo miqyosida bir jinsli bo‘lmagan suyuqliklarning g‘ovak muhitlarda sizishi jarayonlarini tadqiq qilishda bir qator ilmiy natijalar olingan, jumladan, nazariy usullar ishlab chiqilgan (Neft muhandisligi instituti, Germaniya, Amerika neft instituti, AQSH, Yaponiya neft instituti, Yaponiya, Neft qidirish va ishlab chiqarish ilmiy-tadqiqot instituti, Xitoy, Neft va gaz instituti, Rossiya, RFA SB Neft kimyosi instituti, Rossiya, Akademik A. P. Krilov nomidagi Butunrossiya neft-gaz ilmiy-tadqiqot instituti, Rossiya, RFA neft va gaz muammolari instituti, Rossiya, "O‘ZLITINEFTGAZ", O‘zbekiston).

Hozirda dunyoda ichimlik suvlarini tozalash va konlarning neft beruvchanligini oshirish maqsadida ikkilamchi va uchlamchi ishlov berish muammolari nazariyasini ishlab chiqish va muammoni yechishning samarali usullarini topishning ustuvor yo‘nalishlari bo‘yicha tadqiqotlar olib borilmoqda, xususan, bir jinsli bo‘lmagan suyuqliklarning g‘ovak muhitlarda sizishi nazariyasini takomillashtirish va shu asosda ichimlik suvlarini tozalash va neft qazib olishning samarali texnologiyalarini ishlab chiqish bo‘yicha tadqiqotlar o‘tkazilmoqda.

Muammoning o‘rganilganlik darajasi. Jahonda g‘ovak muhitlarda birjinslimas suyuqliklar sizishi masalalari bo‘yicha bir qator olimlar: T.Iwasaki, D.M.Mins, K.Ives, M.Sharma, M.Elimelech, J.Bear, Ch.Tien, P.Bedrikovetsky, A.Adin, M.Rebhun, J.Herzig, V.Gitis, G.Barenblat, Yu.M.Shextman, A.X.Mirzadjanzade, Ye.V.Venetsianov, K.S.Basniyev, D.Mixaylov, A.Nikiforov va boshqa xorijiy olimlar tomonidan tadqiqotlar olib borilgan.

Respublikamizda suyuqlik va gazlarning g‘ovak muhitlarda sizishi jarayonlari matematik modellari va hisoblash usullarini ishlab chiqishda F.B.Abutaliyev, J.F.Fayzullayev, N.M.Muxiddinov, R.Sadullayev, I.Alimov, J.Akilov, B.X.Xo‘jayorov, I.K.Xujayev, N.Ravshanov, Sh.Qayumov, V.F.Burnashev, J.Maxmudov va boshqa olimlarning ulkan hissalari qo‘shilgan.

G‘ovak muhitlarda modda ko‘chishi va suspenziya sizishi jarayonlarini matematik modellashtirish zamonaviy fan va texnikaning eng muhim yo‘nalishlaridan biri hisoblanib, bu jarayonlarni matematik modellashtirishning rivojlanishi bilan innovatsiyalar va texnologik taraqqiyot uchun yangi imkoniyatlar ochilmoqda. Birjinslimas suyuqliklarning g‘ovak muhitlarda sizishi va modda ko‘chishi jarayonida harakatdagi suyuqlik va g‘ovak muhit devorlarining o‘zaro ta’siri murakkab jarayon bo‘lib, bu kabi muammolarni tadqiq qilishda konveksiya, molekulyar diffuziya, dispersiya, o‘tirib qolish kabi hodisalarni hisobga olgan holda balans va kinetika tenglamalarini keltirib chiqarish va ularni tenglamalar sistemasi sifatida birgalikda tahlil qilishni taqozo etadi, bu xususiy hosilali nochiziqli differensial tenglamalar uchun chegaraviy masalalar matematik nuqtai nazardan murakkab ekanligi va bugungi kunda yetarli darajada o‘rganilmaganligini g‘ovak muhitlarda suyuqliklar sizishi va modda ko‘chishi bo‘yicha bajarilgan ilmiy ishlarning tahlilidan kelib chiqadi.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti Sharof Rashidov nomidagi Samarqand davlat universitetining OT-F4-64 «Birjinslimas g‘ovak muhitlarda moddalarning ko‘chishi va suyuqliklarning sizishi gidrodinamik modellarini tuzish va sonli tadqiq etish» (2017-2020) va FZ-

2020092877 «Bir jinsli boʻlmagan gʻovak muhitlarda anomal moddalar koʻchishi va suyuqliklar sizishi jarayonlarining matematik modellarini tuzish va sonli tadqiq etish» (2022-2027) mavzusidagi fundamental ilmiy tadqiqot loyihasi doirasida bajarilgan.

Tadqiqotning maqsadi murakkab kinetikaga olib keluvchi turli omillarni hisobga olib gʻovak muhitlarda modda koʻchishi va suyuqliklar sizishi jarayonlarining matematik modellarini tuzish, sonli tahlil qilish va model parametrlarini identifikatsiyalashdan iborat.

Tadqiqotning vazifalari:

suspenziyalarning gʻovak muhitlarda sizishi masalasini konvektiv koʻchish, gidrodinamik dispersiya, muallaq zarralar oʻtirib qolishi va qayta oqimga qoʻshilishi va boshqalarni hisobga olib modda koʻchishi matematik modelini takomillashtirish va masalalarni sonli yechish;

ikki komponentali suspenziyalarning gʻovak muhitlarda sizishi jarayonining dinamik omillar, diffuziya va koʻp bosqichli kinetikani hisobga olgan holda matematik modelini takomillashtirish va sonli tahlil qilish;

gʻovak muhitlarda modda koʻchishi jarayonining matematik modellari parametrlarini identifikatsiyalashga oid teskari masalalar yechish;

gʻovak muhitlarda modda koʻchishining gibrid modelini tuzish. Bunda keyk qatlam hosil qilib suspenziya suzulishi (cake filtration) matematik modeli muhitning ichki qismida choʻkma hosil boʻlib birjinslimas suyuqliklar sizishi (deep bed filtration) modellari bilan “tikiladi”;

gʻovak muhitlarda modda koʻchishining gibrid modelini tuzish koʻp bosqichli kinetikani hisobga olgan holda takomillashtirish.

Tadqiqotning obyekti gʻovak muhit va unda choʻkma hosil qilib sizuvchi bir jinsli boʻlmagan suyuqlikdan iborat.

Tadqiqotning predmeti gʻovak muhitlarda suspenziyalar sizishi jarayonlarining matematik modellari, hisoblash algoritmlari va gidrodinamik tahlilidan iborat.

Tadqiqot usullari. Dissertatsiyada matematik modellashtirish, tutash muhitlar mexanikasi, hisoblash matematikasi va hisoblash eksperimenti usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

suspenziyalarning gʻovak muhitlarda sizishi modeli koʻp bosqichli kinetika tenglamasi asosida muhit parametrlarining oʻzgaruvchanligi, “eskirish”, “yuklanish” hodisalarini hisobga olgan holda takomillashtirilgan;

ikki komponentali suspenziyalarning gʻovak muhitlarda sizishi jarayonining matematik modeli dinamik omillar, diffuziya va koʻp bosqichli kinetikani hisobga olgan holda takomillashtirilgan;

gʻovak muhitlarda modda koʻchishi jarayonining matematik modellari parametrlarini identifikatsiya qilish algoritmlari ishlab chiqilgan;

gʻovak muhitlarda modda koʻchishining teskari masalalarini yechish uchun parallel hisoblashga asoslangan samarali algoritm ishlab chiqilgan;

gʻovak muhitlarda keyk qatlam hosil qilib suspenziya suzulishi (cake filtration) va muhitning ichki qismida choʻkma hosil boʻlib suspenziya sizishi (deep bed filtration) jarayonlarini bir vaqtning o‘zida hisobga oluvchi gibrid matematik model tuzilgan;

g‘ovak muhitlarda modda ko‘chishining gibrid modeli ko‘p bosqichli kinetika tenglamasi asosida “eskirish”, “yuklanish” hodisalarini hisobga olgan holda takomillashtirilgan.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

g‘ovak muhitlarda modda ko‘chishi jarayonining takomillashtirilgan matematik modellari va hisoblash algoritmlari tutash muhitlar mexanikasi va gidrogeologiya masalalarini yechishda qo‘llanilgan;

bir va ko‘p komponentali suspenziyalarning g‘ovak muhitlarda sizishi masalalarini sonli yechishning samarali algoritmi ishlab chiqilgan;

g‘ovak muhitlarda modda ko‘chishi jarayonining matematik modellarini sonli realizatsiya qilish va parametrlarini identifikatsiya qilish uchun dasturiy majmua yaratilgan.

Tadqiqot natijalarining ishonchliligi. Modda ko‘chishi modellarini tuzishda ko‘p fazali tizimlar uchun fundamental saqlanish qonunlaridan foydalanilgan, ular fizik korrektlikni ta’minlash maqsadida fenomenologik munosabatlar bilan to‘ldirilgan. Moddalarning ko‘chish jarayonida g‘ovak muhitlarda o‘tirib qolishi va qayta suyuqlik oqimiga qo‘shilishi yangi kinetik tenglamasini tuzishda fenomenologik yondashuvdan foydalanilgan va har bir faraz sinchiklab asoslangan. Sonli usullarni qo‘llashda ularning turg‘unligi tekshirilgan va talab qilingan aniqlikdagi approksimatsiyalardan foydalanilgan. Olingan natijalar fizik jihatdan batafsil tahlil qilingan va ularning real fizik jarayonlarga mosligi baholangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati ko‘plab hodisalarni hisobga olgan holda yaratilgan nazariy asos va matematik modellar, sonli usul va algoritmlar g‘ovak muhitlarda modda ko‘chishi nazariyasiga hissa qo‘shishi bilan izohlanadi.

Olingan natijalarning amaliy ahamiyati oqova va ichimlik suvlarini tozalash, neft va gaz konlariga ikkilamchi ishlov berish, gidrogeologiya kabi sohalardagi sizish jarayonlarini sifat va miqdor jihatdan baholashga xizmat qiladi.

Tadqiqot natijalarining joriy qilinishi. Ko‘p bosqichli cho‘kish kinetikasi asosida moddalarning g‘ovak muhitlarda ko‘chishi modellarini tuzish va sonli tadqiq etish uchun ishlab chiqilgan matematik modellar, sonli algoritmlar va dasturiy vositalar asosida:

Malayziya Terengganu universiteti (Universiti Malaysia Terengganu) da bajarilgan UMT/CRIM/2- 2/2/14 Jld. 4(44), Vote No 55233 – sonli “Modified Homotopy Analysis Method for Solving System of Linear and Nonlinear Integral Equations and modelling on tension and scaling factor of the power cable” mavzusidagi xorijiy loyihani bajarishda ushbu dissertatsiya natijalaridan foydalanilgan (16.01.2024 – yil UMT/FSKM/500-45 – son ma’lumotnomasi). Fizika va muhandislikdagi ko‘plab masalalarning yechimini sonli hisoblash uchun chekli ayirmali sxemalarga asoslangan sonli algoritmlar va xususiy hosilali differensial tenglamalar sistemasini yechish uchun ishlab chiqilgan dasturiy ta’minotdan foydalanilgan. Yuqoridagi algoritmdan foydalanib, giperbolik sistemalarda qaytariladigan tenglama uchun model masalalarining sonli yechimi olingan. Dissertatsiyada ishlab chiqilgan dasturiy ta’minotdan foydalanish sonli yechimning model masalalarining aniq yechimiga yaqinlashishini asoslaydigan va natijalarni vizualizatsiya qiladigan hisoblash tajribalarini oʻtkazish imkonini bergan;

Oʻzbekiston Respublikasi Fanlar akademiyasi Mexanika va inshootlar seysmik mustahkamligi institutida 2022–2024-yillarda bajarilgan № IL-21071166–sonli “Shamolning past tezligi uchun moʻljallangan vertikal oʻqli shamol turbinasini yaratish” mavzusidagi innovatsion loyiha doirasida qoʻllanilgan (2026-yil 11-mart, 331-3-sonli ma’lumotnoma). Xususan, shamol oqimining past tezliklardagi barqarorligini modellashtirishda reaksiya–diffuziya tipidagi tenglamalardan foydalanib matematik modellar tuzilgan. Hosil qilingan tenglamalar koeffitsiyentlarini har doim ham oshkor ravishda topib boʻlmaganligi sababli, dissertatsiyada ishlab chiqilgan xususiy hosilali differensial tenglamalar uchun koeffitsiyentli teskari masalalarni yechish usul va algoritmlariga oid natijalar ushbu modellar parametrlarini topishda qoʻllanilgan. Natijada shamolning kichik tezliklarida samarali faoliyat koʻrsatadigan, vertikal aylanish oʻqi va harakatlanuvchi ishchi elementlarga ega shamol energiya qurilmasining optimal parametrlarini aniqlash imkoniyati yaratilgan;

gʻovak muhitlarda ikki komponentali suspenziya sizishi takomillashtirilgan modeliga oid ilmiy natijalardan yetakchi xorijiy jurnallarda (International Journal of Non-Linear Mechanics, Volume 167, December 2024, 104905, Annali di Matematica Pura ed Applicata, Volume 2012, May 2022, 2943–2964, Applied Mathematics and Mechanics, Volume 45, January 2024, 355–372) birjinslimas suyuqliklarning gʻovak muhitlarda sizishi masalalarini tadqiq qilishda foydalanilgan. Ilmiy natijalarning qoʻllanilishi qaralayotgan masalalarning adekvat modellarini tuzish imkonini bergan;

gʻovak muhitlarda suspenziya sizishi masalasini sonli yechish algoritmiga oid ilmiy natijalardan yetakchi xorijiy jurnallarda (Journal of Applied Engineering Science, Volume 19, January 2021, 768, Geoenergy Science and Engineering,Volume 235, April 2024, 212696, Journal of Physics: Conference Series, Volume 1926, January 2021, 012001) ba’zi nochiziqli differensial tenglamalarga mos masalalarni tadqiq qilishda foydalanilgan. Ilmiy natijalarning qoʻllanilishi qaralayotgan masalalarning sonli yechimini olish imkonini bergan.

Tadqiqot natijalarining aprobasiyasi. Mazkur tadqiqot natijalari, 34 ta ilmiy-amaliy anjumanlarda, jumladan, 25 ta xalqaro va 9 ta respublika ilmiy-amaliy anjumanlarida muhokamadan oʻtkazilgan.

Tadqiqot natijalarining e’lon qilinishi. Dissertatsiya mavzusi boʻyicha jami 53 ta ilmiy ish chop etilgan, shulardan, Oʻzbekiston Respublikasi Fanlar Akademiyasi huzuridagi Oliy Attestatsiya komissiyasining doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 13 ta maqola, jumladan, 10 tasi xorijiy va 3 tasi respublika jurnallarida nashr etilgan. Xorijiy jurnallarda chop etilgan 10 ta maqolaning hammasi Scopus bazasida indekslangan jurnallarda, shu jumladan 4 tasi Q1-Q2 kvartilga kiruvchi jurnallarda chop etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya tarkibi kirish, to‘rtta bob, xulosa, foydalanilgan adabiyotlar ro‘yxati va ilovalardan iborat. Dissertatsiyaning hajmi 165 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurligi asoslangan, dissertatsiya mavzusi bo‘yicha xorijiy ilmiy tadqiqotlar sharhi berilgan, tadqiqotning maqsadi, vazifalari, obyekti va predmeti, uning respublika fan va texnikasi rivojlanishining ustuvor yo‘nalishlari bilan muvofiqligi bayon qilingan, tadqiqotning ilmiy yangiligi va amaliy natijalari berilgan, tadqiqot natijalarining ishonchliligi, ilmiy va amaliy ahamiyati, chop etilgan ishlar va dissertatsiya tuzilishi keltirilgan.

Dissertatsiyaning birinchi bobida g‘ovak muhitlarda modda ko‘chishi va suspenziya sizishi jarayonlari, keyk qatlam hosil qilib suspenziya suzulishi (cake filtration) va muhitning ichki qismida cho‘kma hosil bo‘lib birjinslimas suyuqliklar sizishi (deep bed filtration) jarayonlarini modellashtirishda foydalanilgan yondashuvlar haqida qisqacha ma’lumotlar keltirilgan. Shu vaqtgacha qo‘llanilgan yondashuvlarning umumiy xarakterli xususiyatlari aniqlangan. Xususan, modellashtirishning fenomenologik yondashuvlari (modda balansi va boshqa) tenglamalaridan, shuningdek, umumlashgan Darsi qonunidan foydalanishga asoslangan. Shuningdek, bu modellar asosida tuzilgan to‘g‘ri va teskari masalalarni sonli yechish usullari sharhlangan.

Dissertatsiyaning “Ko‘p bosqichli cho‘kma kinetikasi asosida g‘ovak muhitda suspenziya sizishini matematik modellashtirish” deb nomlangan ikkinchi bobida ko‘p bosqichli cho‘kma kinetikasini hisobga olgan holda g‘ovak muhitda suspenziya sizishining to‘g‘ri masalalari qaralgan. Ushbu jarayonlarni matematik modellashtirish oqimning gidrodinamik parametrlari, g‘ovak muhitning tuzilishi va suspenziya tarkibidagi muallaq zarrachalarning o‘tirib qolishi va qayta suyuqlik oqimiga qo‘shilishi mexanizmlari kabi bir qator omillarni hisobga olishni talab qiladi.

Faraz qilaylik, ma’lum bir $t > 0$ vaqtdan boshlab, toza (muallaq zarrachalarsiz) suyuqlik bilan to‘yingan, boshlang‘ich g‘ovakligi m_0 va L uzunlikdagi g‘ovak muhitga o‘zgarmas $v(t) = v = const$ tezlik bilan c_0 konsentratsiyali suspenziya kirishi boshlansin. Bir o‘lchovli holatda suspenziya oqimidagi muallaq zarralarning massa balansi tenglamasi quyidagi shaklga ega

$$m_0 \frac{\partial c}{\partial t} + v \frac{\partial c}{\partial x} + \frac{\partial \rho}{\partial t} = D \frac{\partial^2 c}{\partial x^2}, \qquad (1)$$

bu yerda c - suspenziya konsentratsiyasi, v - sizish tezligi (m/s), m_0 - boshlang‘ich g‘ovaklik, ρ - cho‘kma konsentratsiyasi, D - diffuziya koeffitsiyenti, x – koordinata, t - vaqt.

Cho‘kish kinetika tenglamasi quyidagi shaklda olinadi

$$\frac{\partial\rho}{\partial t}=\begin{cases}\beta_1 vc, & 0<\rho\le\rho_1,\\ \beta_2 vc-\beta_3(1+\gamma|\nabla p|)\rho, & \rho_1<\rho\le\rho_2,\\ \beta_2\frac{\rho_0}{\rho}vc-\beta_3(1+\gamma|\nabla p|)\rho, & \rho_2<\rho<\rho_0,\\ 0, & \rho=\rho_0,\end{cases} \tag{2}$$

bu yerda, ρ_0 – filtrning umumiy sig‘imi, ρ_1 - ρ ning “yuklanish” yakunlanadigan qiymati, β_1 - “yuklanish” effekti bilan bog‘liq kinetika koeffitsiyenti, β_2 - zarrachalarning cho‘kishi bilan bog‘liq koeffitsiyent, β_3 - zarrachalarning qayta suyuqlik oqimiga qo‘shilishi bilan bog‘liq koeffitsiyent, $|\nabla p|$ - bosim gradiyenti moduli, γ - bosim gradiyentining zarrachalarning qayta suyuqlik oqimiga ta’sirini ifodalovchi o‘zgarmas koeffitsiyent.

$|\nabla p|$ ni topish uchun Darsi qonunidan foydalanamiz

$$v=K(m)|\nabla p|, \tag{3}$$

bu yerda $K(m)$ sizish koeffitsiyenti, $m=m_0-\rho$ joriy g‘ovaklik.

$K(m)$ ni topish uchun Karmen- Kozeni tenglamasidan foydalanamiz

$$K(m)=k_0m^3/(1-m)^2,\ k_0=const. \tag{4}$$

Boshlang‘ich va chegaraviy shartlar quyidagi shaklda olinadi

$$c(0,t)=c_0=const,\ \frac{\partial c}{\partial x}=0,\quad x=L,\quad t>0,\ c(x,0)=0,\quad \rho(x,0)=0. \tag{5}$$

(1)-(5) masala chekli ayirmalar usuli yordamida yechildi. $D=\{0\le x\le L,\ 0\le t\le T\}$ sohada to‘r kiritamiz. Buning uchun biz $[0,L]$ intervalni h qadam bilan va $[0,T]$ ni τ qadam bilan bo‘laklarga ajratamiz, bu yerda T sizish jarayoni o‘rganilayotgan vaqt. Natijada quyidagi to‘rga ega bo‘lamiz

$$\omega_{h\tau}=\left\{(x_i,t_j),\ x_i=ih,\ i=0,1,...I,\ t_j=j\tau,\ j=0,1,...,J,\quad \tau=T/J\right\}.$$

$c(t,x)$, $\rho(t,x)$ funksiyalar o‘rniga (x_i,t_j) tugunlardagi qiymatlari mos ravishda $c_i^j\,.\rho_i^j$ bo‘lgan to‘r funksiyalari kiritiladi.

(1)-(4) tenglamalar $\omega_{h\tau}$ to‘rda quyidagicha approksimatsiya qilinadi

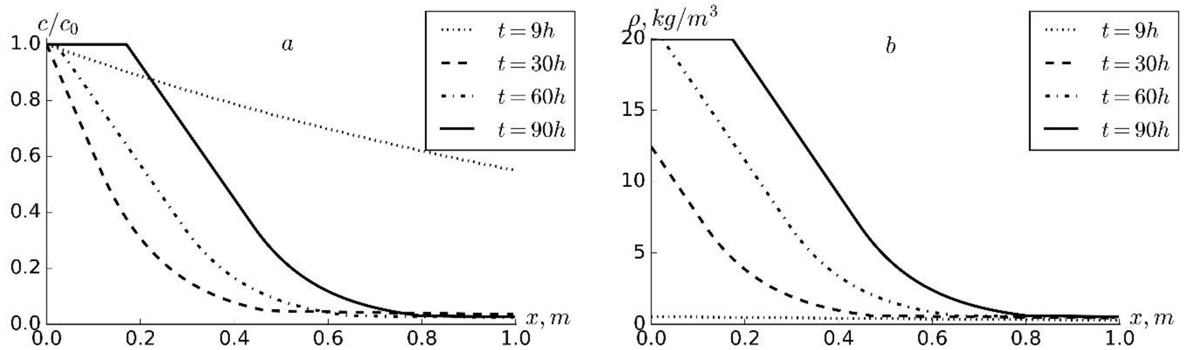
$$m_0\frac{c_i^{j+1}-c_i^j}{\tau}+v_0\frac{c_i^{j+1}-c_{i-1}^{j+1}}{h}+\frac{\rho_i^{j+1}-\rho_i^j}{\tau}=D\frac{c_{i-1}^{j+1}-2c_i^{j+1}+c_{i+1}^{j+1}}{h^2}, \tag{6}$$

$$\frac{\rho_i^{j+1}-\rho_i^j}{\tau}=\begin{cases}\beta_1 vc_i^j, & 0<\rho_i^j\le\rho_1,\\ \beta_2 vc_i^j-\beta_3\left(1+\gamma|\nabla p|_i^{j+1}\right)\rho_i^j, & \rho_1<\rho_i^j\le\rho_2,\\ \beta_2\frac{\rho_0}{\rho_i^j}vc_i^j-\beta_3\left(1+\gamma|\nabla p|_i^{j+1}\right)\rho_i^j, & \rho_2<\rho_i^j<\rho_0,\\ 0\,, & \rho_i^j=\rho_0,\end{cases}, \tag{7}$$

$$\left|\nabla p\right|_i^{j+1}=\frac{v_0\left(1-m_0+\left(\rho_{a,i}^j+\rho_{n,i}^j\right)\right)^2}{k_0\left(m_0-\left(\rho_{a,i}^j+\rho_{n,i}^j\right)\right)^3}. \tag{8}$$

(5) boshlang‘ich va chegaraviy shartlari ham approksimatsiya qilinadi

$$c_i^0=0,\ \rho_i^0=0,\ i=\overline{0,I},\ c_0^j=c_0,\ j=\overline{0,J},\ c_I^j-c_{I-1}^j=0,\ j=\overline{0,J}. \tag{9}$$



1-rasm. c/c_0 (a), ρ (b) profillari, bu yerda $\rho_1=0,6$, $\rho_2=7$.

1-rasmda turli vaqt momentlarida c/c_0 muallaq zarrachalarning nisbiy konsentratsiyasi va ρ cho‘kma konsentratsiyasining profillari ko‘rsatilgan. $t=9h$ (1.a-rasm) da c/c_0 profilining sezilarli oldinga siljishi kuzatiladi, bu zarrachalar cho‘kishining deyarli to‘liq yo‘q bo‘lgan davriga to‘g‘ri keladi. Buni, "yuklanish" bosqichida cho‘kish intensivligi juda past bo‘lishi bilan izohlash mumkin. Vaqt o‘tishi bilan zarrachalar cho‘kishi jarayoni butun filtr zonasini qoplaydi va maksimal qiymatigacha yetadi.

Ikki komponentli suspenziyalarning matematik modeli bir komponentli suspenziya uchun modelning umumlashtirish natijasida hosil qilinadi. Zarrachalar massasi balansi tenglamasi har bir komponenta uchun yoziladi, bu bir o‘lchovli shakldа quyidagi shaklda ifodalanadi

$$m_0\frac{\partial c^{(i)}}{\partial t}+v\frac{\partial c^{(i)}}{\partial x}+\frac{\partial \rho^{(i)}}{\partial t}=D^{(i)}\frac{\partial^2 c^{(i)}}{\partial x^2}, \tag{10}$$

bu yerda $c^{(i)}$ - suspenziya i - komponenti tarkibidagi muallaq zarrachalar konsentratsiyalari, $\rho^{(i)}$ - i- komponentning cho‘kma konsentratsiyalari, $D^{(i)}$ - i-komponent uchun diffuziya koeffitsiyenti (m^2/s), $i=1,2$, komponent raqamlariga mos keladi.

Kinetik tenglamalarni quyidagi shaklda olamiz

$$\frac{\partial \rho^{(1)}}{\partial t}=\begin{cases}\beta_r^{(1)} v c^{(1)}, & 0<\rho^{(1)}\le\rho_1^{(1)},\\ \dfrac{\beta_a^{(1)} v c^{(1)}}{1+\gamma^{(1)}\left|\nabla p\right|}-\beta_d^{(1)}\left(1+\omega^{(1)}\left|\nabla p\right|\right)\rho^{(1)}, & \rho_1^{(1)}<\rho^{(1)},\ \rho^{(1)}+\rho^{(2)}<\rho_0,\\ 0, & \rho^{(1)}+\rho^{(2)}=\rho_0,\end{cases} \tag{11}$$

$$\frac{\partial\rho^{(2)}}{\partial t}=\begin{cases}\beta_r^{(2)}vc^{(2)}, & 0<\rho^{(2)}\le\rho_1^{(2)},\\ \dfrac{\beta_a^{(2)}vc^{(2)}}{1+\gamma^{(2)}|\nabla p|}-\beta_d^{(2)}\left(1+\omega^{(2)}|\nabla p|\right)\rho^{(2)}, & \rho_1^{(2)}<\rho^{(2)}<\rho_0^{(2)},\rho^{(1)}+\rho^{(2)}<\rho_0,\\ 0, & \rho^{(2)}=\rho_0^{(2)}\text{ or }\ \rho^{(1)}+\rho^{(2)}=\rho_0,\end{cases}\quad(12)$$

bu yerda $\gamma^{(i)}$, $\omega^{(i)}$ doimiy koeffitsiyentlardir.

Bir o‘lchovli holatda Darsi qonuni quyidagi shaklga ega

$$v=K(m)|\nabla p|,\ m=m_0-\left(\rho^{(1)}/\rho_d^{(1)}+\rho^{(2)}/\rho_d^{(2)}\right),\quad(13)$$

bu yerda $K(m)$ sizish koeffitsiyenti, $\rho_d^{(i)}$ zarrachalar materiallarining zichligi, kg/m^3.

Boshlang‘ich va chegara shartlari quyidagi shaklga ega

$$c^{(i)}(x,0)=0,\ \ \rho^{(i)}(x,0)=0,\quad(14)$$

$$c^{(i)}(0,t)=c_0^{(i)}=\text{const},\ \frac{\partial c^{(i)}}{\partial x}=0,\ \ x=L,\ \ t>0.\quad(15)$$

(10) - (15) masala chekli ayirmalar usuli bilan yechilgan.

Passiv zonalarning sig‘imi uchun $\rho_0^{(1)}=15$ va $\rho_0^{(2)}=5$ ishlatilgan. Muhitning kirish qismiga yaqin joyda, vaqt o‘tishi bilan suspenziyaning birinchi komponentining konsentratsiyasi birinchi komponent uchun muhitning sig‘imidan kattaroq bo‘ladi, ya’ni $\rho^{(1)}>\rho_0^{(1)}$. Shu sababli, ikkinchi komponent uchun passiv zonaning sig‘imi nafaqat $\rho^{(2)}$ konsentratsiyasi bilan , balki $\rho^{(1)}$ bilan ham to‘yingan. Taxminan $t=30h$ da, ikkinchi komponentning cho‘kma konsentratsiyasi x ning ma’lum qiymatigacha biroz oshadi va keyin kamayadi. Vaqt o‘tishi bilan $\rho^{(1)}$ ning ortish oralig‘i kengayadi.

Endi, boshlang‘ich g‘ovakliligi m_0 bo‘lgan, birjinsli suyuqlik bilan to‘ldirilgan yarim cheksiz birjinsli g‘ovak muhitni qaraymiz. $x=0$ nuqtada $c_0=c_0^{(1)}+c_0^{(2)}$konsentratsiyali ikki komponentli suspenziya ($c_0^{(1)}$, $c_0^{(2)}$ mos ravishda kiruvchi suspenziyasidagi birinchi va ikkinchi turdagi zarrachalarning konsentratsiyalari) $v(t)=v=const$ tezlik bilan g‘ovak muhitga yuboriladi.

Diffuziya hadini o‘z ichiga olgan berilgan o‘zgarmas oqim tezligi rejimiga ega bo‘lgan balans tenglamasi quyidagicha:

$$m\frac{\partial c^{(i)}}{\partial t}+v\frac{\partial c^{(i)}}{\partial x}+\frac{\partial\rho_a^{(i)}}{\partial t}+\frac{\partial\rho_n^{(i)}}{\partial t}=D^{(i)}\frac{\partial^2c^{(i)}}{\partial x^2}\quad(16)$$

bu yerda m muhitning joriy g‘ovakliligi, $c^{(i)}$ suspenziyaning i - komponentining konsentratsiyasi, $\rho_a^{(i)}$, mos ravishda $\rho_p^{(i)}$suspenziyaning i - komponentining aktiv va passiv zonalarida cho‘kma konsentratsiyasi, $D^{(i)}$ har bir komponent uchun diffuziya koeffitsiyenti, $i=1,2$ komponentlar soniga mos keladi.

Ikki komponentli suspenziya uchun aktiv zonada zarrachalarning o‘tirib qolishi va qayta oqimga qo‘shilishi jarayonining kinetik tenglamasi quyidagicha umumlashtirildi.

$$\frac{\partial \rho_a^{(i)}}{\partial t} = \frac{\beta_a^{(i)}}{1+\gamma^{(i)}\left|\nabla p\right|} c^{(i)} - \beta_a^{(i)} \frac{\rho_a^{(i)}\left(1+\omega^{(i)}\left|\nabla p\right|\right)}{\rho_{a0}^{(i)}} c_0^{(i)} \tag{17}$$

bu yerda $i=1,2.$, $\rho_{a0}^{(i)}$ - suspenziyaning *i*-komponenti uchun aktiv zonalarning sig‘imi, $\beta_a^{(i)}$ suspenziyaning *i*-komponenti uchun aktiv zonasidagi kinetikani tavsiflovchi koeffitsiyent, $\gamma^{(i)}$, $\omega^{(i)}$ aktiv zonada o‘tirib qolishi va qayta oqimga qo‘shilishi jarayoniga bosim gradientining ta’sir intensivligini tavsiflovchi doimiy koeffitsiyentlardir.

Passiv zonada qayta suyuqlik oqimiga qo‘shilmaydigan cho‘kma hosil bo‘lishi kinetikasining tenglamalari quyidagicha

$$\frac{\partial \rho_p^{(i)}}{\partial t} = \beta_p^{(i)} \phi_i\left(\rho_p^{(1)}, \rho_p^{(2)}\right) c^{(i)} \tag{18}$$

bu yerda $\beta_p^{(i)}$ suspenziyaning *i*-komponentining passiv zonasidagi kinetikasini koeffitsiyent.

Cho‘kmaning siqilishi (“eskirishi”) ta’sirini tavsiflovchi parametrlar quyidagi tenglamalar bilan ifodalanadi $\phi_i\left(\rho_p^{(1)}, \rho_p^{(2)}\right)$

$$\phi_1\left(\rho_p^{(1)}, \rho_p^{(2)}\right) = \begin{cases} 1, & 0 < \rho_p^{(1)} \le \rho_{p1}^{(1)}, \\ \rho_{p1}^{(1)} \big/ \rho_p^{(1)}, & \rho_{p1}^{(1)} < \rho_p^{(1)},\ \rho_p^{(1)} + \rho_p^{(2)} < \rho_{p0}, \\ 0, & \rho_p^{(1)} + \rho_p^{(2)} = \rho_{p0}, \end{cases} \tag{19}$$

$$\phi_2\left(\rho_p^{(1)}, \rho_p^{(2)}\right) = \begin{cases} 1, & 0 < \rho_p^{(2)} \le \rho_{p1}^{(2)}, \\ \rho_{p1}^{(2)} \big/ \rho_p^{(2)}, & \rho_{p1}^{(2)} < \rho_p^{(2)} < \rho_{p0}^{(2)},\ \rho_p^{(1)} + \rho_p^{(2)} < \rho_{p0}, \\ 0, & \rho_p^{(2)} = \rho_{p0}^{(2)}\ or\ \rho_p^{(1)} + \rho_p^{(2)} = \rho_{p0}, \end{cases} \tag{20}$$

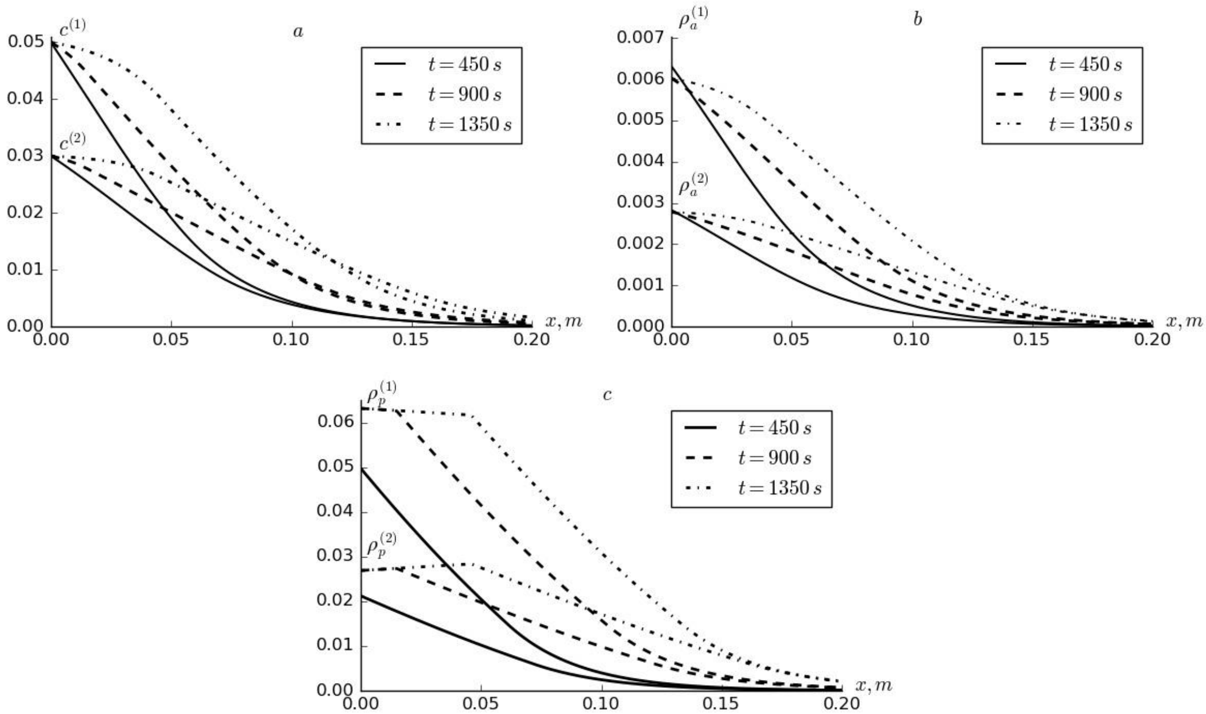
bu yerda $\rho_{p1}^{(i)}$ - suspenziyaning mos keladigan komponenti uchun cho‘kmaning siqilishini (“eskirishi”) boshlaydigan chegaraviy konsentratsiyalar, ya’ni $\rho_p^{(i)} > \rho_{p1}^{(i)}$ dan boshlab “eskirish” hodisasi kuzatiladi.

Boshlang‘ich va chegaraviy shartlar quyidagicha

$$c^{(i)}(x,0) = 0, \quad \rho_a^{(i)}(x,0) = \rho_p^{(i)}(x,0) = 0,\ c^{(i)}(0,t) = c_0^{(i)} = \text{const}. \tag{21}$$

(16)-(21) masalasini yechish uchun biz chekli ayirmalar usulidan foydalanamiz.

2-rasmda $c^{(i)}$, $\rho_a^{(i)}$ va $\rho_p^{(i)}, (i=1,2)$ o‘zgarishlar profillari dinamik omillar va diffuziya effektlari bilan ko‘rsatilgan. Vaqt o‘tishi bilan qatlamdagi fiksirlangan nuqtalarda $c^{(i)}$, $\rho_a^{(i)}$va $\rho_p^{(i)}$ qiymatlari oshadi (2-rasm). Passiv zonalarning sig‘imi uchun $\rho_{p0}^{(1)}=0,06$, $\rho_{p0}^{(2)}=0,03$ ishlatilgan. 2c-rasmdan ko‘rinib turibdiki, muhit kirish joyiga yaqin joyda, vaqt o‘tishi bilan suspenziyaning birinchi komponentining konsentratsiyasi birinchi komponenta uchun passiv zonaning sig‘imidan kattaroq bo‘ladi, ya’ni $\rho_p^{(1)} > \rho_{p0}^{(1)}$. Shu sababli, ikkinchi komponent uchun passiv zonaning sig‘imi nafaqat konsentratsiya $\rho_p^{(2)}$ bilan, balki $\rho_p^{(1)}$ bilan ham to‘yingan.



2-rasm. $c^{(1)}, c^{(2)}$ (a), $\rho_a^{(1)}, \rho_a^{(2)}$ (b), $\rho_p^{(1)}, \rho_p^{(2)}$ (c) larning profillari, bu yerda

$$\rho_{p1}^{(1)}=0,030, \rho_{p1}^{(2)}=0,015,\ \gamma^{(1)}=\omega^{(1)}=0.1; \gamma^{(2)}=\omega^{(2)}=0.05,$$

$$D^{(1)}=2.0\cdot 10^{-6}\, m^2/s,\ D^{(2)}=1.5\cdot 10^{-6}\, m^2/s.$$

Dinamik omillar hisobga olinmagan holda masalalar qaralganda, har bir komponent uchun aktiv zonaning sig‘imi $\rho_{a_0}^{(i)}$, *x=0* nuqtada cho‘kma bilan to‘liq to‘yinganligini kuzatishimiz mumkin. Dinamik omillar hisobga olinganda esa, *x=0* nuqtada vaqt o‘tishi bilan avval $\rho_a^{(i)}$ ortadi, keyin kamayadi va nihoyat aktiv zonaning sig‘imi cho‘kma bilan to‘liq to‘yinmasdan oldin dinamik muvozanat yuzaga keladi.

Ko‘p komponentli suspenziya sizishining matematik modeli bitta komponentli holatning umumlashtirilishi bo‘lib, suspenziyaning har bir komponenti uchun massa balansi tenglamasi quyidagichadir ($i=1,2$)

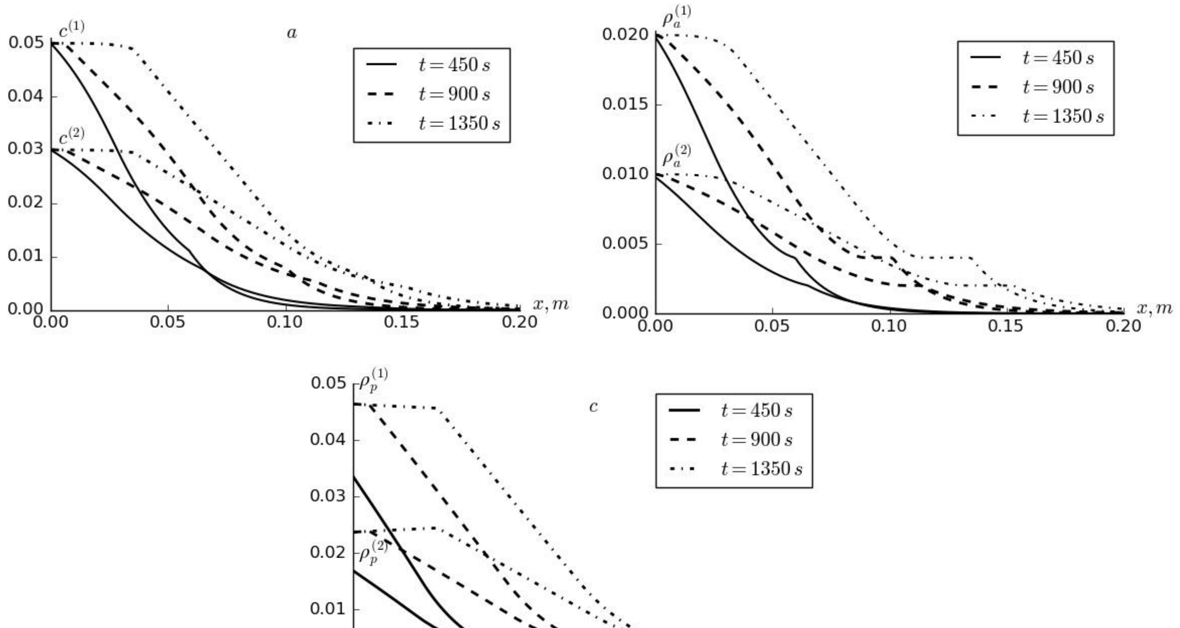
$$\frac{\partial\left(mc^{(i)}\right)}{\partial t}+v\frac{\partial c^{(i)}}{\partial x}+\frac{\partial\rho_a^{(i)}}{\partial t}+\frac{\partial\rho_p^{(i)}}{\partial t}=D^{(i)}\frac{\partial^2 c^{(i)}}{\partial x^2}, \tag{22}$$

Aktiv zona uchun kinetika tenglamasi ko‘p bosqichlilikni hisobga olib quyidagicha takomillashtirildi

$$\frac{\partial\rho_a^{(i)}}{\partial t}=\begin{cases}\beta_r^{(i)}vc^{(i)}, & 0<\rho_a^{(i)}\le\rho_{ar}^{(i)},\\ \beta_a^{(i)}vc^{(i)}-\beta_d^{(i)}\rho_a^{(i)}, & \rho_{ar}^{(i)}<\rho_a^{(i)}<\rho_{a0}^{(i)},\\ 0, & \rho_a^{(i)}=\rho_{a0}^{(i)},\end{cases} \tag{23}$$

bu yerda $\beta_r^{(i)}$ - “yuklanish” hodisasi bilan bog‘liq kinetika tenglamasi, $\beta_a^{(i)}$, $\beta_d^{(i)}$ - mos ravishda muallaq zarrachalarning o‘tirib qolishi va qayta suyuqlik oqimiga qo‘shilishi intensivligini ifodalovchi kinetika koeffitsiyentlari.

3b-rasm natijalari shuni ko‘rsatadiki, faol zonadagi “yuklanish” hodisasining ta’siri konsentratsiyaning o‘zgarish profillariga sezilarli darajada ta’sir qiladi. Parametr oshgani sayin ikkita zona hosil bo‘ladi: biri $\rho_a^{(i)}\le\rho_{ar}^{(i)}$ bilan, ikkinchisi esa $\rho_a^{(i)}>\rho_{ar}^{(i)}$. Bu zonalarda filtrlash xususiyatlari turlicha o‘zgarish tezligiga ega. Aktiv zonadagi cho‘kma konsentratsiyasining o‘zgarish tezligi passiv zonadagi o‘tirib qolgan zarrachalar konsentratsiyasiga va cho‘kma xususiyatlariga ham ta’sir qiladi.



3-rasm. $c^{(1)}, c^{(2)}$ (a), $\rho_a^{(1)},\rho_a^{(2)}$ (b), $\rho_p^{(1)},\rho_p^{(2)}$ (c) ning $\rho_{ar}^{(1)}=0{,}004$, da joylashgan profillari $\rho_{ar}^{(2)}=0{,}002$.

“G‘ovak muhitda suspenziyani sizishining teskari masalalari” deb nomlangan 3-bobda model parametrlarini aniqlash bo‘yicha teskari masalalar shakllantirilgan va sonli yechilgan.

Quyidagi masalani qaraymiz: uzunligi L bo‘lgan, boshlang‘ich g‘ovakligi m_0 bo‘lgan, toza (muallaq zarrachalarsiz) suyuqlik bilan to‘yingan birjinsli g‘ovak muhitga, ma’lum $t>0$ vaqtdan boshlab, c_0 konsentratsiyali suspenziya $v(t)=v_0=\text{const}$ tezlik bilan kirishi boshlansin. U holda masalaning matematik modelini quyidagi tenglamalar sistemasi bilan ifodalash mumkin

$$m_0\frac{\partial c}{\partial t}+v_0\frac{\partial c}{\partial x}+\frac{\partial \rho}{\partial t}=D\frac{\partial^2 c}{\partial x^2}, \tag{24}$$

$$\frac{\partial \rho}{\partial t}=\begin{cases}\beta_r v_0 c, & 0<\rho\le\rho_r,\\ \beta_a v_0 c-\beta_d\rho, & \rho_r<\rho\le\rho_0,\\ 0, & \rho=\rho_0,\end{cases} \tag{25}$$

bu yerda c - suspenziya konsentratsiyasi (kg/m^3), ρ - cho‘kma konsentratsiyasi (kg/m^3), ρ_0 - filtrning umumiy sig‘imi (kg/m^3), D - diffuziya (yoki dispersiya) koeffitsiyenti, ρ_r - ρ ning shunday qiymatiki, shu qiymatda “yuklanish” hodisasi to‘xtaydi, β_r - “yuklanish” bosqichida muallaq zarrachalarning o‘tirib qolish intensivligini belgilovchi kinetika koeffitsiyenti, β_a - asosiy bosqichda muallaq zarrachalarning o‘tirib qolish intensivligini belgilovchi kinetika koeffitsiyenti, β_d - asosiy bosqichda o‘tirib qolgan zarrachalarning qayta suyuqlik oqimiga qo‘shilishini ifodalovchi kinetika koeffitsiyenti.

Boshlang‘ich va chegaraviy shartlarni quyidagicha olamiz

$$\begin{aligned}&c(0,t)=c_0=\text{const},\\ &\frac{\partial c}{\partial x}=0,\quad x=L,\quad t>0,\\ &c(x,0)=0,\\ &\rho(x,0)=0.\end{aligned} \tag{26}$$

(24) - (26) masala g‘ovak muhitda suspenziya sizishining masalasining to‘g‘ri qo‘yilishiga mos keladi. Bunda (24), (25) da barcha koeffitsiyentlar ma’lum, shuningdek, (26) da c_0 ham berilganda $c(x,t),\rho(x,\mathrm{t})$ yechimlar topiladi.

Teskari masalani yechishda, (24), (25) da ba’zi koeffitsiyentlar noma’lum va aniqlanishi kerak bo‘ladi. Bu masalani yechish to‘g‘ri masalaning yechimi haqida qo‘shimcha ma’lumot talab qiladi. Buning uchun biz dastlab parametrlarning ma’lum qiymatlari bilan to‘g‘ri masalani yechamiz. Qo‘shimcha ma’lumot sifatida biz $Z_l(t)$ deb belgilaydigan ba’zi berilgan $x_l\in(0,L)$, $l=\overline{1,n}$ nuqtalardagi muallaq zarrachalar konsentratsiyasi qiymatlaridan foydalanamiz.

Buning uchun kvazi-real tajriba o‘tkazamiz, ya’ni (24)-(26) to‘g‘ri masalani $\beta_r^{exact}=0.8$ 1/m, $\beta_a^{exact}=7.5$ 1/m ma’lum bo‘lgan holda yechamiz. Sonli hisoblash natijalariga ko‘ra, $z_l^j=z_l(t_j)$, $j=0,1,...,M$ funksiya topiladi. Hisoblash tajribalari $z(t)$ funksiya tasodifiy xatoliklar yordamida quyidagi tarzda qo‘zg‘atilgan hollarda ham amalga oshiriladi:

$$\left(z_l^j\right)_\delta = z_l^j + 2\delta\left(\sigma^j - \frac{1}{2}\right),$$

bu yerda σ^j tasodifiy funksiya bo‘lib, u $[0;\ 1]$oraliq bo‘ylab tekis taqsimlangan, δ xatolik darajasidir. Qo‘zg‘atilgan $z_l(t)$ funksiyani quyidagicha belgilaymiz $z_l^\delta(t)$.

To‘g‘ri masalaning yechimi haqida qo‘shimcha ma’lumotlardan foydalangan holda (25) tenglamaning β_r va β_a koeffitsiyentlarini aniqlashdan iborat bo‘lgan suspenziya sizishining teskari masalasini ko‘rib chiqamiz. Masalani yechish uchun biz identifikatsiya usulidan foydalanamiz. Teskari masalani quyidagicha qo‘yamiz: β_r va β_a va koeffitsiyentlarini quyidagi funksional uchun minimum shartidan aniqlansin

$$\Phi(\beta_r,\beta_a) = \sum_{l=1}^{n}\int_0^T\left[c(x_l,t) - z_l(t)\right]^2 dt, \tag{27}$$

(27) funksional uchun statsionarlik sharti quyidagi shaklga ega

$$\frac{d\Phi(\boldsymbol{\gamma})}{d\boldsymbol{\gamma}} = 2\cdot\sum_{l=1}^{n}\int_0^T\left[c(x_l,t) - z_l(t)\right]\cdot\omega(z_l,t)dt = 0, \tag{28}$$

bu yerda ω vektor ustun, γ vektor satr,

$$\frac{d\rho}{d\boldsymbol{\gamma}} = \omega = (\omega_{11},\omega_{12})^T, \quad \boldsymbol{\gamma} = (\beta_r,\beta_a), \tag{29}$$

bu yerda $\omega_{11} = \dfrac{\partial c}{\partial\beta_r}, \omega_{12} = \dfrac{\partial c}{\partial\beta_a}$ - β_r va β_a koeffitsiyentlariga nisbatan sezgirlik funksiyalari.

c funksiyasini γ^s atrofida ikkinchi darajali hadlargacha yoyamiz

$$c^{s+1}(x,t) \approx c^s(x,t) + \left(\boldsymbol{\gamma}^{s+1} - \boldsymbol{\gamma}^s\right)\cdot\omega^s(x,t), \tag{30}$$

(30) yoyilmani (27) munosabatga qo‘yib, β_r^{s+1}, β_a^{s+1} lar uchun quyidagi chiziqli algebraik tenglamalar sistemasini olamiz

$$\begin{cases} a_{11}\beta_r^{s+1} + a_{12}\beta_a^{s+1} = b_1, \\ a_{21}\beta_r^{s+1} + a_{22}\beta_a^{s+1} = b_2, \end{cases} \tag{31}$$

bu yerda

$$a_{11} = \sum_{l=1}^{n}\int_0^T \omega_{11}^{s\,2}(x_l,t)dt,$$

$$a_{22} = \sum_{l=1}^{n}\int_0^T \omega_{12}^{s\,2}(x_l,t)dt,$$

$$a_{12} = a_{21} = \sum_{l=1}^{n}\int_0^T \omega_{11}^{s\,2}(x_l,t)\cdot\omega_{12}^{s\,2}(x_l,t)dt, \tag{32}$$

$$b_1=\sum_{l=1}^{n}\int_0^T\left[\omega_{11}^s(x_l,t)\beta_r^s+\omega_{12}^s(x_l,t)\beta_a^s-c^s(x_l,t)+z_l(t)\right]\omega_{11}^s(x_l,t)dt\ ,$$

$$b_2=\sum_{l=1}^{n}\int_0^T\left[\omega_{11}^s(x_l,t)\beta_r^s+\omega_{12}^s(x_l,t)\beta_a^s-c^s(x_l,t)+z_l(t)\right]\omega_{12}^s(x_l,t)dt\ ,$$

(24)-(26) ni β_r bo‘yicha differensiallab quyidagi masalani olamiz

$$m_0\frac{\partial\omega_{11}}{\partial t}+v_0\frac{\partial\omega_{11}}{\partial x}+\frac{\partial\omega_{21}}{\partial t}=D\frac{\partial^2\omega_{11}}{\partial x^2}, \tag{33}$$

$$\frac{\partial\omega_{21}}{\partial t}=\begin{cases}v_0c+\beta_r v_0\omega_{11}, & 0<\rho\le\rho_r,\\ \beta_a v_0\omega_{11}-\beta_d\omega_{21}, & \rho_r<\rho\le\rho_0,\\ 0, & \rho=\rho_0,\end{cases} \tag{34}$$

$$\omega_{11}(x,0)=0,\quad \omega_{21}(x,0)=0,\quad \omega_{11}(0,\mathrm{t})=0,\qquad \left.\frac{\partial\omega_{11}}{\partial x}\right|_{x=L}=0\,. \tag{35}$$

bu yerda $\omega_{21}=\dfrac{\partial\rho}{\partial\beta_r}$ - β_r koeffitsiyentga nisbatan sezgirlik funksiyasi.

(24)-(26) ni β_a ga nisbatan differensiallab quyidagi masalani olamiz

$$m_0\frac{\partial\omega_{12}}{\partial t}+v_0\frac{\partial\omega_{12}}{\partial x}+\frac{\partial\omega_{22}}{\partial t}=D\frac{\partial^2\omega_{12}}{\partial x^2}, \tag{36}$$

$$\frac{\partial\omega_{22}}{\partial t}=\begin{cases}\beta_r v_0\omega_{12}, & 0<\rho\le\rho_r,\\ v_0c+\beta_a v_0\omega_{12}-\beta_d\omega_{22}, & \rho_r<\rho\le\rho_0,\\ 0, & \rho=\rho_0,\end{cases} \tag{37}$$

$$\omega_{12}(x,0)=0,\quad \omega_{22}(x,0)=0,\quad \omega_{12}(0,\mathrm{t})=0,\qquad \left.\frac{\partial\omega_{12}}{\partial x}\right|_{x=L}=0\,. \tag{38}$$

bu yerda $\omega_{21}=\dfrac{\partial\rho}{\partial\beta_a}$ - β_a koeffitsiyentga nisbatan sezgirlik funksiyasi.

β_r va β_a koeffitsiyentlarini aniqlash algoritmi quyidagicha tuzilishi mumkin:

1. Qandaydir boshlang‘ich yaqinlashishlarni tanlaymiz $\beta_r=\beta_r^0$ va $\beta_a=\beta_a^0$ (bunda, $s=0$).

2. $t=0$ dan $t=T$ gacha (24)-(26), (33)-(35) va (36)-(38) masalalarni yechish va $c^s(x,t)$, $\rho^s(x,t)$, $\omega_{11}^s(x,t)$, $\omega_{21}^s(x,t)$, $\omega_{12}^s(x,t)$, $\omega_{22}^s(x,t)$ funksiyalarini hisoblash

3. (31)-(32) sistemasini yechish va β_r^{s+1}, β_a^{s+1} yaqinlashishlarni topish;

4. Quyidagilarni o‘rnatish $s=s+1$, $\beta_r=\beta_r^{s+1}$, $\beta_a=\beta_a^{s+1}$

5. Quyidagi shart bajarilmaguncha 2, 3, 4-bosqichlarni takrorlash

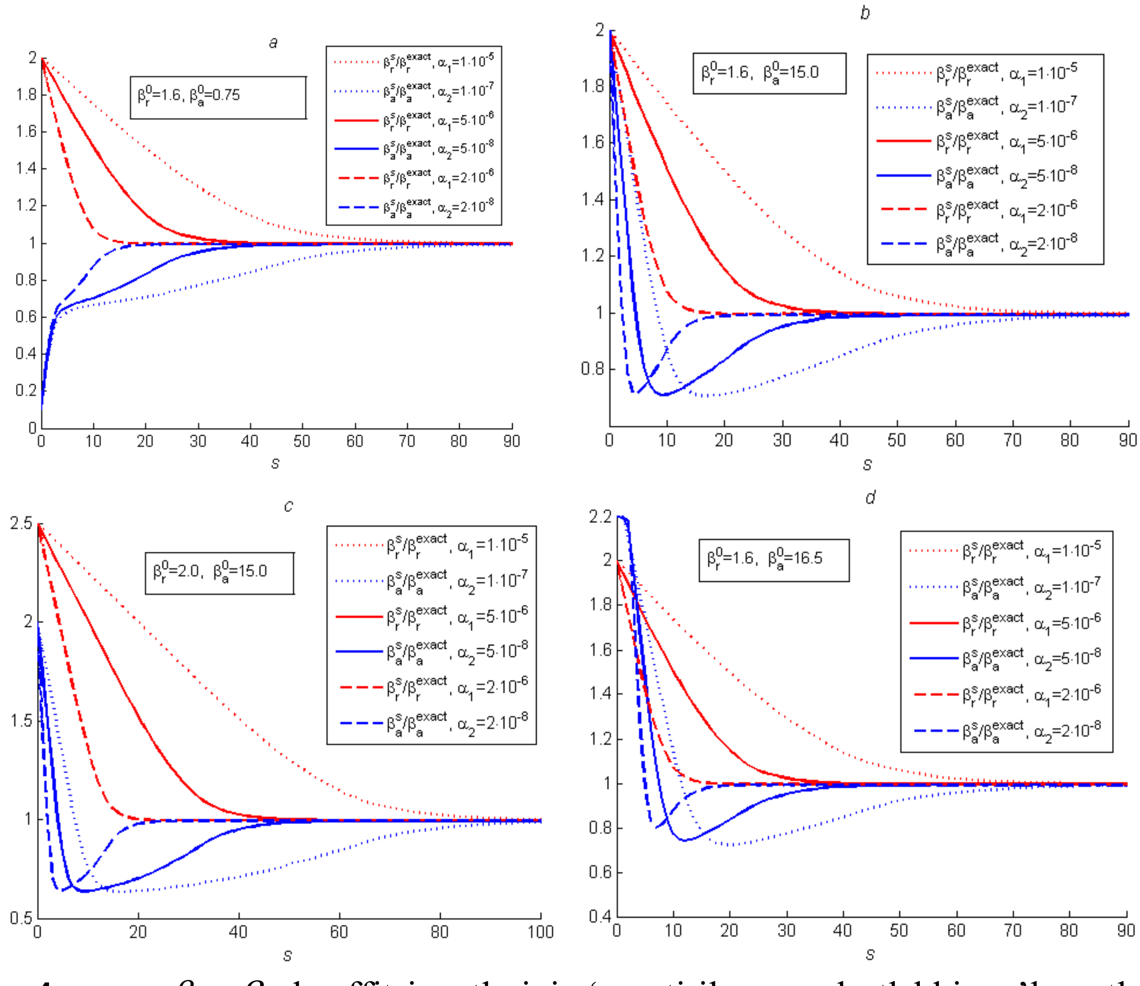
$$\left|\frac{\Phi^{s+1}-\Phi^s}{\Phi^s}\right|\le\varepsilon\,,\qquad \left|\frac{\beta_r^{s+1}-\beta_r^s}{\beta_r^s}\right|\le\varepsilon_1\,,\qquad \left|\frac{\beta_a^{s+1}-\beta_a^s}{\beta_a^s}\right|\le\varepsilon_2\,.$$

bu yerda ε, $\varepsilon_1, \varepsilon_2$ - masala yechimining xatoligini aniqlaydigan yetarlicha kichik sonlar.

β_r va β_a koeffitsiyentlarni aniqlash uchun va turli β_r^0, β_a^0 boshlang‘ich yaqinlashuvlar uchun va muvozanat nuqtasi yaqinida o’tkazilgan hisob-kitoblar natijalari shuni ko‘rsatadiki, hatto muvozanat nuqtasi yaqinida ham yaqinlashuv divergent bo‘ladi (so‘nmas tebranishlar sodir bo‘ladi). Shuning uchun teskari masalani yechish uchun biz o‘zgartirilgan birinchi tartibli usuldan foydalanamiz. Buning uchun biz (27) funksionalni boshqa shakldagi almashtiramiz

$$\Phi(\gamma) = \sum_{l=1}^{n} \int_0^T \left[c(x_l, t) - z_l(t) \right]^2 dt + a\left(\gamma^{s+1} - \gamma^s\right), \qquad (39)$$

bu yerda $\alpha = (\alpha_1, \alpha_2)^T$ – regulyarlashtiruvchi parametrlardan iborat ustun vektor, α_1, α_2 - mos ravishda β_r va β_a uchun regulyarlashtiruvchi parametrlar.

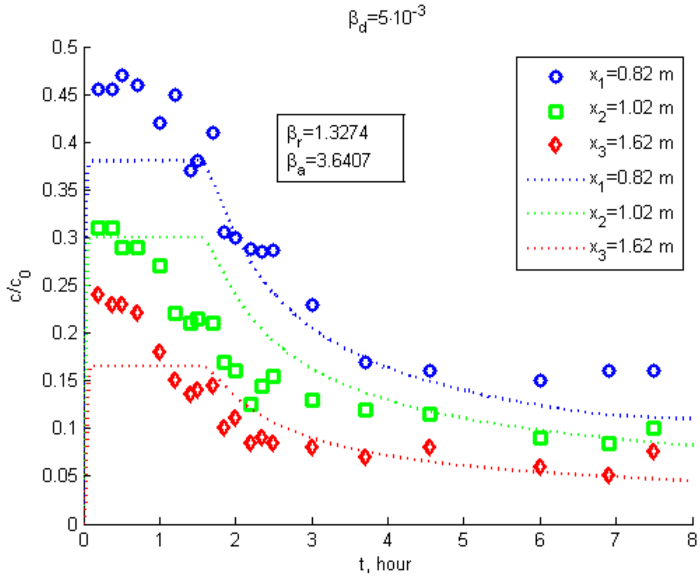


4-rasm. β_r, β_a koeffitsiyentlarini o‘zgartirilmagan dastlabki ma’lumotlar ($\delta = 0$), bilan tiklash β_r^{exact}, β_a^{exact} – rasmdagi kabi.

4-rasmda α_1, α_2 regulyatsiya parametrlari va β_r^0, β_a^0 boshlang‘ich yaqinlashishlarning turli qiymatlariga mos β_r va β_a koeffitsiyentlarni tiklash natijalari keltirilgan. Hisoblash natijalari shuni ko‘rsatadiki, β_r^0, β_a^0 dastlabki yaqinlashuvlar muvozanat nuqtasidan uzoqlashgan sari iteratsiyalar soni ortadi

(4-rasm a◦, b, c, d). Iteratsiyalar soni β_r^0, β_a^0 dastlabki yaqinlashuvlarga qarab yigirma dan yuzgacha o‘zgaradi. Natijalar, shuningdek, α_1, α_2 regulyatsiya parametrlari kamayishi bilan iteratsiyalar soni kamayishini ko‘rsatadi.

Yuqorida keltirilgan algoritmdan foydalanib, biz laboratoriya tajribalaridan olingan dastlabki ma’lumotlar bilan teskari masalani ham yechamiz. Natijalar 5-rasmda keltirilgan. Natijalar qoniqarli ekanligini ko‘rish mumkin.



5-rasm. Tajribalar uchun eksperimental ma’lumotlar (nuqtalar) va model bashorati (egri chiziqlar)

Suspenziya sizishi aktiv va passiv zonalarga ega bo‘lgan g‘ovak muhitlarda sodir bo‘lganda, ya’ni cho‘kma hosil bo‘lishi mos ravishda qaytariladigan va qaytarilmaydigan shakllarga ega. Bu holda, jarayonning matematik modeli ikkala zona uchun ham massa balansi tenglamasi va kinetik tenglamalardan iborat.

$$\frac{\partial c}{\partial t}+U\frac{\partial c}{\partial x}+\frac{d_m}{m}\frac{\partial \rho_a}{\partial t}+\frac{d_m}{m}\frac{\partial \rho_p}{\partial t}=D_L\frac{\partial^2 c}{\partial x^2}, \quad (40)$$

$$\frac{d_m}{m}\frac{\partial \rho_a}{\partial t}=\beta_a c-\beta_d\frac{d_m}{m}\rho_a, \quad (41)$$

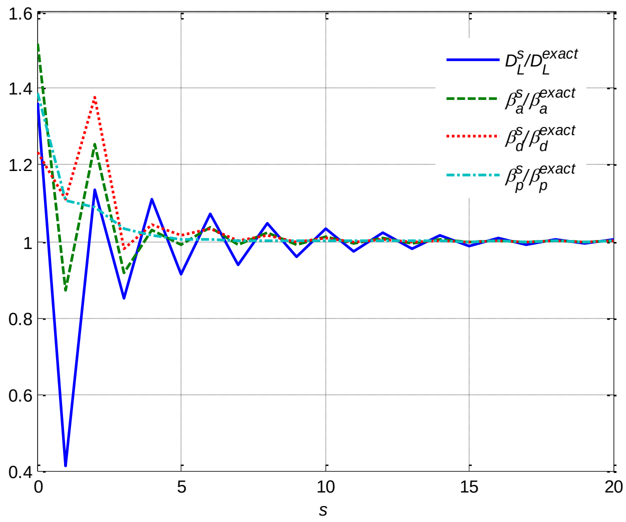
$$\frac{d_m}{m}\frac{\partial \rho_p}{\partial t}=\beta_p c, \quad (42)$$

bu yerda c-suyuqlikdagi muallaq zarrachalar konsentratsiyasi, d_m-cho’kma zichligi, m-g‘ovaklik, ρ_a va ρ_p mos ravishda aktiv va passiv zonalarda cho‘kadigan zarrachalar konsentratsiyasi, D_L-diffuziya koeffitsiyenti, β_a-qaytariladigan cho‘kish tezligi koeffitsiyenti, β_d-qaytariladigan cho‘kish qayta suyuqlikka qo‘shilishi koeffitsiyenti, β_p-qaytarilmaydigan cho‘kish tezligi koeffitsiyenti, x-koordinata.

Boshlang‘ich va chegaraviy shartlar quyidagicha:

$$c(x,0)=0,\quad \rho_a(x,0)=\rho_p(x,0)=0, c(0,t)=\begin{cases} c_0, & 0\le t\le t_1 \\ 0, & t>t_1 \end{cases},\quad \left.\frac{\partial c}{\partial t}\right|_{x=L}=0. \quad (43)$$

(40)-(43) tenglamalardagi $D_L, \beta_a, \beta_d, \beta_p$ koeffitsiyentlarini aniqlashdan iborat teskari masalani ko‘rib chiqamiz .



6-rasm. Muvozanat nuqtasi atrofida $D_L, \beta_a, \beta_d, \beta_p$ koeffitsiyentlarini tiklash.

Biz ushbu koeffitsiyentlarni quyidagi funksionalni minimallashtirish orqali aniqlaymiz

$$\Phi\left(D_L, \beta_a, \beta_d, \beta_p\right) = \int_0^T \left[c\left(L,t\right) - z\left(t\right)\right]^2 dt \qquad (44)$$

Teskari masalani yechish uchun yuqoridagi algoritm asosida sonli hisob-kitoblar amalga oshirildi. 6-rasmda koeffitsiyentlar muvozanat nuqtasi atrofida boshlang‘ich qiymatlar muvozanat nuqtasiga yaqin bo‘lgan holda qanday tiklanganligi ko‘rsatilgan. Hisoblash natijalari shuni ko‘rsatadiki, barcha parametrlar yetarli aniqlik bilan tiklangan. Muvozanat nuqtasiga yaqin boshlang‘ich qiymatlarga ega koeffitsiyentlarni tiklash uchun 6-10 tagacha iteratsiya kerak bo‘ldi. Muvozanat nuqtasiga yaqin boshlang‘ich qiymatlarga ega koeffitsiyentlarni tiklash uchun 8-20 tagacha iteratsiya kerak bo‘ldi.

G‘ovak muhitdagi ko‘p bosqichli kinetikaga asoslangan suspenziya sizishi modelining parametrlarini aniqlashda parallel hisoblash algoritmi ishlab chiqildi. Parallel hisoblashning asosiy maqsadi masalani yechish vaqtini qisqartirishdir. Ishlab chiqilgan parallel algoritm yordamida kompyuterda ham tajriba o‘tkazildi. Biz hisob-kitoblarni faqat to‘rt va sakkiz yadroli noutbuklarda amalga oshirdik. Ushbu algoritmdan foydalanish bizga 4 yadroli noutbuklardan foydalanganda vaqtni 1,7 baravar, 8 yadroli kompyuterlardan foydalanganda esa 2,6 baravar tejash imkonini berdi.

“G‘ovak muhitda suspenziya sizishining gibrid modellarini ishlab chiqish” deb nomlangan IV bobda gibrid model ishlab chiqilgan. Filtr yuzasida keyk qatlami hosil qilish bilan suspenziyalar sizishi (cake filtration) va zarrachalarning cho‘kishi muhit ichida hosil bo‘ladigan g‘ovak muhitlarda birjinslimas suyuqliklarning sizishi (deep bed filtration) kombinatsiyalangan jarayonini o‘rganish uchun bizga zarrachalarning filtr muhiti ichida va yuzasida ko‘chishi va cho‘kishini tavsiflovchi adekvat modellar kerak.

Uzunligi s_0, boshlang‘ich g‘ovakliligi φ_p bo‘lgan, dastlab toza (zarrachasiz) suyuqlik bilan to‘yingan g‘ovak muhitga ma’lum bir $t>0$ vaqtdan boshlab o‘zgarmas $u=const$ tezlikda C_{in} qattiq zarrachalar konsentratsiyasiga ega bo’lgan suspenziya kira boshlaydi.

Filtr yuzasida keyk qatlam hosil qilib suspenziyalar sizishi va g‘ovak muhitda chuqur qatlamli sizish o‘rtasida kiruvchi zarrachalar oqimini ajratish uchun keyk qatlamining o‘sish koeffitsiyenti tushunchasi - λ kiritiladi. Bu yerda λ o‘lchovsiz parametr (0≤ λ ≤1), ya’ni kiruvchi zarrachalar konsentratsiyasining λ qismi darhol muhit yuzasiga cho‘kib, keyk qatlamini hosil qiladi, qolgan $1-\lambda$ qismi muhitga kiradi va quyi qatlamlarda filtrlanadi.

Jarayonning matematik modeli, keyk qatlam uchun balans va kinetika tenglamasidan, shuningdek, bir o‘lchovli holatda quyidagi shakldа ifodalangan keyk qatlamining o‘sish tenglamasidan iborat.

$$\frac{\partial C_p}{\partial t}+u\frac{\partial C_p}{\partial x}-D_p\frac{\partial^2 C_p}{\partial x^2}+\frac{\partial M_p}{\partial t}=0,\quad x\in[0,s_0], \tag{45}$$

$$\frac{\partial M_p}{\partial t}=k_p u\left(C_p-\frac{(1-\lambda)\mathrm{C}_{in}}{M_{p0}}M_p\right),\quad x\in[0,s_0], \tag{46}$$

$$\frac{\partial C_c}{\partial t}+u\frac{\partial C_c}{\partial t}-D_c\frac{\partial^2 C_c}{\partial x^2}+\frac{\partial M_c}{\partial t}=0,\quad x\in[s_0,\mathrm{s(t)}], \tag{47}$$

$$\frac{\partial M_c}{\partial t}=k_c u\left(C_c-\frac{(1-\lambda)\mathrm{C}_{in}}{M_{c0}}M_c\right), x\in[s_0,\mathrm{s(t)}], \tag{48}$$

$$\frac{ds}{dt}=\lambda\frac{C_{in}|u|}{(1-\varphi_c)},\ s(0)=s_0,\ x=s(t), \tag{49}$$

$$\frac{dM_S}{dt}=\lambda|u|C_{in},\qquad x=s(t), \tag{50}$$

bu yerda C_p, C_c-mos ravishda suspenziyaning g‘ovak muhit va keyk qatlamidagi konsentratsiyalari, m^3/m^3, u -sizish tezligi (m/s), D_p, D_c-diffuziya koeffitsiyentlari, m^2/s, M_p, M_c-cho‘kma konsentratsiyalari, m^3/m^3, M_s-sirt konsentratsiyasi, k_p, k_c-cho‘kma hosil bo‘lish intensivligini ifodalovchi kinetik koeffitsiyentlar, M_{p0}, M_{c0} hosil bo‘lgan cho‘kmaning maksimal qiymati, sig‘im, φ_c-keyk g‘ovakligi, p,c - indekslar mos ravishda g‘ovak muhit va keyk qatlamini ifodalaydi.

Harakatlanuvchi $x=s(t)$ sirtda (keyk qatlamining yuqori qismida) C_c chegara sharti o‘rnatiladi. Harakatlanuvchi chegara modelida zarrachalarning ma’lum bir λ qismi darhol keykga olib tashlangani sababli, faqat $1-\lambda$ kiruvchi konsentratsiya keykga oqayotgan suspenziya sifatida kiradi. Bu C_c sirtdagi kirish shartini o‘zgartirish orqali amalga oshirildi.

$$uC_c - D_c \frac{\partial C_c}{\partial x} = \left(1-\lambda\right)uC_{in}, \quad x = s(t). \tag{51}$$

Chap chegarada biz quyidagi shartdan foydalanamiz

$$\frac{\partial C_p}{\partial x} = 0, \qquad x = 0. \tag{52}$$

G‘ovak muhitning keyk bilan chegarasida quyidagi shartlardan foydalanamiz

$$C_p = C,\ x = s_0, \tag{53}$$

$$uC_p - D_p \frac{\partial C_p}{\partial x} = uC_c - D_c \frac{\partial C_c}{\partial x}, \quad x = s_0. \tag{54}$$

(53) shartni hisobga olgan holda, (54) uchun quyidagilarni yozishimiz mumkin:

$$D_p \frac{\partial C_p}{\partial x} = D_c \frac{\partial C_c}{\partial x}. \tag{55}$$

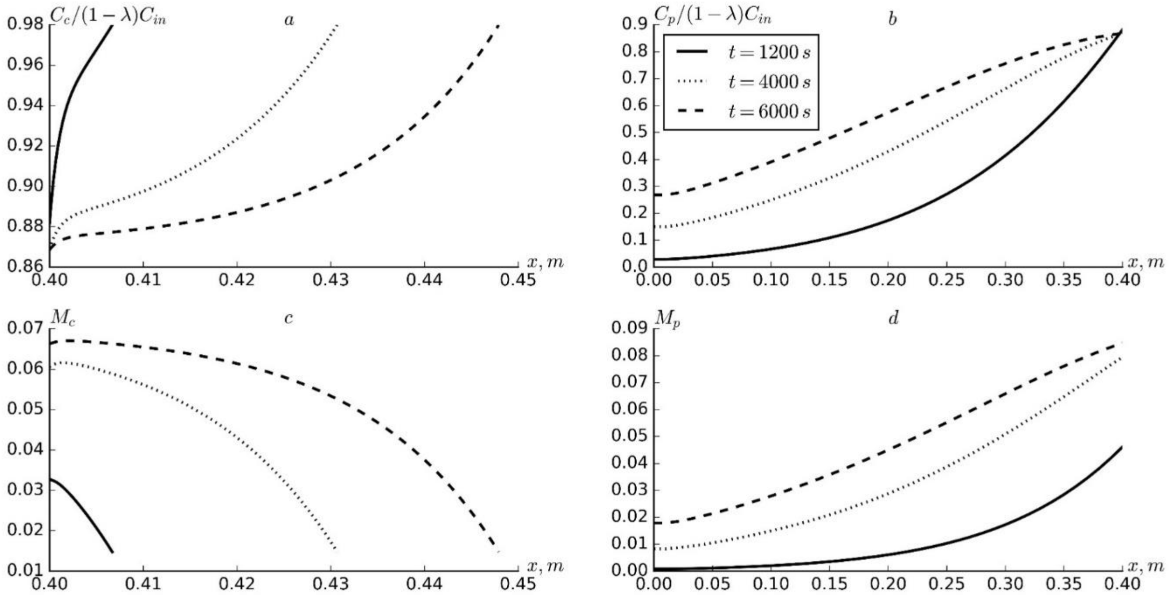
Bu sistemani quyidagi boshlang‘ich shartlar ostida yechamiz

$$C_p(0,x) = 0, \tag{56}$$

$$C_c(0,x) = 0. \tag{57}$$

7-rasmda $C_c, C_p, M_c, M_p,$ konsentratsiya profillari ko‘rsatilgan. 7a-rasmdan ko‘rinib turibdiki, keyk qatlamdagi muallaq zarrachalar konsentratsiyasi cho‘kma qatlam qalinligi s ortishi bilan o‘zgaradi. Yuqorida ta’kidlanganidek, C_{in} kiruvchi konsentratsiyaning faqat $(1-\lambda)$ qismi keyk qatlami bo‘ylab taqsimlanadi. Biroq, konsentratsiya qiymati maksimal $C_c = (1-\lambda)C_{in}$ qiymatiga erisha olmaydi, bu (51) shart bilan izohlanadi. Chunki $x = s(t)$ bu harakatlanuvchi chegara bo‘lib, belgilangan nuqtalarda C_c konsentratsiya chegaraga yaqin nuqtalarda yuqoriroq bo‘ladi, ammo vaqt o‘tishi bilan, bu konsentratsiyaning bir qismi g‘ovak muhitga o‘tganda, belgilangan nuqtalardagi konsentratsiya ma’lum bir qiymatgacha pasayadi. 7b-rasmda g‘ovak muhitda muallaq zarrachalar konsentratsiyasining taqsimlanishi ko‘rsatilgan va ko‘rinib turibdiki, keyk qatlami va g‘ovak muhit chegarasida kiruvchi konsentratsiya qiymati turlicha bo‘lishi mumkin. Biroq, g‘ovak muhitning boshqa nuqtalarida konsentratsiya asta-sekin oshib boradi. 7c- rasmda keyk qatlamidagi o‘tirib qolgan zarrachalar konsentratsiyasi ko‘rsatilgan va uning qiymati vaqt o‘tishi bilan ortib borishini ko‘rish mumkin.

7.c-rasmdagi M_p va 7.d-rasmdagi M_c larning o‘zgarishlarini taqqoslash orqali quyidagi xulosaga kelish mumkin: $x = s_0$ nuqta atrofida berilgan vaqtlarda g‘ovak muhitdagi konsentratsiya C_p ning qiymati C_c ning qiymatidan kichik bo‘lsa ham, M_p ning qiymatlari M_c dan kattaroq ekanligini ko‘rish mumkin. Bu g‘ovak muhitda cho‘kma hosil bo‘lish intensivligi keyk qatlamiga qaraganda yuqoriroq ekanligi bilan izohlanadi.



7-rasm. $C_c(a)$, $C_p(b)$, $M_c(c)$, $M_p(d)$ profillari $D_c = 10^{-6}$ m²/s va $D_p = 10^{-5}$ m²/s.

Shuningdek, qo‘yilgan masala uchun tuzilgan matematik model ko‘p bosqichli cho‘kma kinetikasi holatiga umumlashtirildi.

(46), (46) cho‘kma kinetikasi quyidagi shaklda olinadi

$$\frac{\partial M_p}{\partial t} = \begin{cases} k_{rp}uC_p, & 0 \le M_p \le M_{p1}, \\ k_{ap}uC_p - k_{dp}M_p, & M_{p1} < M_p < M_{p0}, \\ 0, & M_p = M_{p0}, \end{cases} \tag{58}$$

$$\frac{\partial M_c}{\partial t} = \begin{cases} k_{rc}uC_c, & 0 \le M_c \le M_{c1}, \\ k_{ac}uC_c - k_{dc}M_c, & M_{c1} < M_c < M_{c0}, \\ 0, & M_c = M_{c0}. \end{cases} \tag{59}$$

Kinetika tenglamalarida ko‘p bosqichlilik xususiyatini hisobga olish C_c, C_p, M_c, M_p konsentratsiyalar profillariga ma’lum darajada ta’sir ko‘rsatadi. Grafiklarning silliqligi yomonlashgani va burilishlar paydo bo‘lganini ko‘rish mumkin. Vaqtning kichik qiymatlarida belgilangan nuqtalarda hosil bo‘lgan cho‘kma konsentratsiyalari bir bosqichli bo‘lgan holga qaraganda kamroq, ammo katta vaqt qiymatlari uchun teskari manzarani kuzatish mumkin. Bu "yuklanish" effektining ta’siri bilan izohlanadi.

Uzunligi s_0, dastlab toza (zarrachasiz) suyuqlik bilan to‘yingan g‘ovak muhitga ma’lum bir $t > 0$ vaqtdan boshlab o‘zgarmas $v = const$ tezlikda konsentratsiyasi $c_0 = c_{10} + c_{20}$ bo‘lgan, har xil o‘lchamdagi ikki turdagi qattiq zarrachalarni o‘z ichiga olgan birjinslimas suyuqlik filtrga kira boshlaydi. Suspenziyadagi yirik zarrachalar g‘ovak muhit yuzasiga cho‘kadi va cho‘kma qatlam (keyk) hosil qiladi, mayda zarrachalar esa g‘ovak muhitga kirib, chuqur qatlamli filtrlash jarayonida ishtirok etadi. Shuningdek, mayda zarrachalar keyk qatlamida ham cho‘kishi kuzatiladi.

G‘ovak muhitda sizish tenglamalari

$$m_2\frac{\partial c_2}{\partial t}+v\frac{\partial c_2}{\partial x}+\frac{\partial \sigma_2}{\partial t}=D_2\frac{\partial^2 c_2}{\partial x^2},\qquad x\in\left[0,s_0\right], \tag{60}$$

$$\frac{\partial \sigma_2}{\partial t}=k_{2a}vc_2-k_{2d}\sigma_2,\qquad x\in\left[0,s_0\right], \tag{61}$$

bu yerda c_2-suspenziyadagi muallaq zarrachalarining konsentratsiyasi, m_2-g‘ovak muhitning g‘ovaklik koeffitsiyenti, σ_2-g‘ovak muhitda hosil bo‘lgan cho‘kma konsentratsiyasi, D_2-g‘ovak muhitda diffuziya koeffitsiyenti , k_{2a}-g‘ovak muhitda qattiq zarrachalarning cho‘kishini tavsiflovchi koeffitsiyent, k_{2d}-g‘ovak muhitda qattiq zarrachalarning teskari oqimga oqib chiqishini tavsiflovchi koeffitsiyent.

Keyk qatlamining o‘sishini tavsiflovchi tenglama quyidagicha yoziladi:

$$\frac{ds}{dt}=\frac{c_1 v}{\left(1-m_1\right)},\quad x=s(t), \tag{62}$$

bu yerda c_1 - suspenziyadagi yirik zarrachalar konsentratsiyasi, $s\left(t\right)$ - keyk qatlamining qalinligi, m_1 - keyk qatlamining g‘ovaklik koeffitsiyenti.

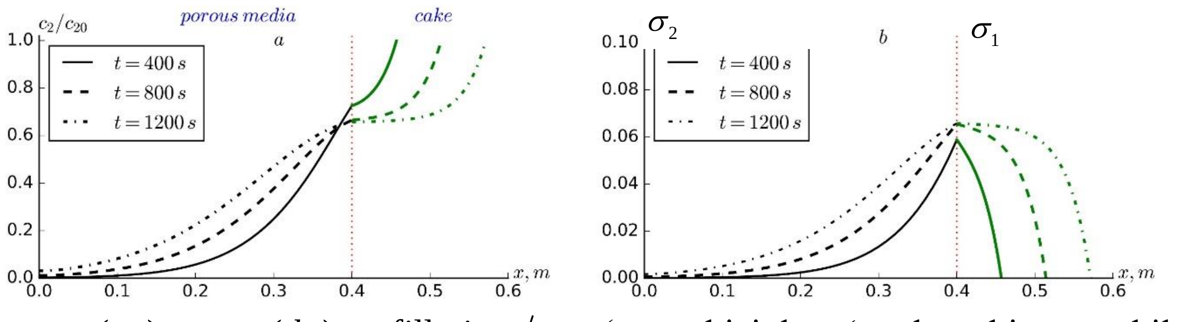
Keyk qatlamidagi sizish tenglamalari quyidagicha:

$$m_1\frac{\partial c_2}{\partial t}+v\frac{\partial c_2}{\partial t}+\frac{\partial \sigma_1}{\partial t}=D_1\frac{\partial^2 c_2}{\partial x^2},\qquad x\in\left[s_0,s\left(t\right)\right], \tag{63}$$

$$\frac{\partial \sigma_1}{\partial t}=k_{1a}vc_2-k_{1d}\sigma_1,\quad x\in\left[s_0,s\left(t\right)\right], \tag{64}$$

bu yerda σ_1 - keyk qatlamida cho‘kma hosil qiluvchi mayda zarrachalar konsentratsiyasi, D_1-keyk qatlamidagi diffuziya koeffitsiyentidir, k_{1a} – keyk qatlamida mayda qattiq zarrachalarning cho‘kishini tavsiflovchi koeffitsiyent, k_{1d} – qattiq zarrachalarning keyk qatlamidagi oqimga qaytishini tavsiflovchi koeffitsiyent.

8-rasmda g‘ovak muhitda filtrlashning gibrid matematik modelining sonli yechimidan olingan natijalar ko‘rsatilgan. Grafiklarda keyk qatlami va g‘ovak muhitdagi muallaq zarrachalar konsentratsiyasi, shuningdek, cho‘kkan zarrachalar konsentratsiyasi ko‘rsatilgan, bunda g‘ovak muhitdagi zarrachalar konsentratsiyasi qora rangda va keyk qatlamida yashil rangda ko‘rsatilgan.



8-rasm. (*a*), σ_1,σ_2(*b*) profillari c_2/c_{20} . (qora chiziqlar g‘ovak muhitga, yashil chiziqlar cho‘kma qatlamga, ko‘ndalang qizil nuqta chiziq cho‘kma qatlami va g‘ovak muhit o‘rtasidagi chegarani ifodalaydi).

Natijalar tahlili shuni ko‘rsatadiki, vaqt o‘tishi bilan keyk qatlamining qalinligi oshadi va keyk qatlamida ham, g‘ovak muhitda ham muallaq zarrachalar konsentratsiyasi va cho‘kma konsentratsiyasi mutanosib ravishda oshadi. Belgilangan nuqtalarda, frontning oldinga siljishi tufayli vaqt o‘tishi bilan keyk qatlamida muallaq zarrachalar konsentratsiyasining taqsimlanishining pasayishi kuzatiladi. G‘ovak muhitning bu xususiyati faqat $s_0 = 0,4$ nuqtaga yaqin nuqtalarda saqlanib qoladi va undan uzoqlashganda, aksincha, vaqt o‘tishi bilan konsentratsiyaning ortishi kuzatilishi mumkin. Cho‘kma konsentratsiyasida ham g‘ovak muhitda, ham cho‘kma qatlamida vaqt o‘tishi bilan faqat o‘sish kuzatiladi.

XULOSALAR

Tadqiqotning asosiy natijalari quyidagilar:

1. Cho‘kma hosil bo‘lishining barcha bosqichlarini hisobga olgan holda, ko‘p bosqichli cho‘kma kinetikasiga asoslangan g‘ovak muhitda bir komponentali suspenziya uchun matematik model ishlab chiqildi.
2. Ikki zonali g‘ovak muhitda ikki komponentali suspenziya sizishi matematik modeli g‘ovak muhit xususiyatlaridagi o‘zgarishlar va ko‘p bosqichli cho‘kma kinetikasini hisobga olgan holda takomillashtirildi.
3. G‘ovak muhitda suspenziya sizishi modelining parametrlarini aniqlash uchun koeffitsiyentli teskari masala qo‘yildi va sonli yechildi. Masalani yechish uchun qo‘llanilgan identifikatsiya usuli avval qoniqarli natijalar bermasligi aniqlandi. Shuning uchun birinchi tartibli regulyarlashtirilgan identifikatsiya usuli qo‘llanildi. Regulyarlashtirish parametrining yetarlicha kichik qiymatlari uchun kinetik koeffitsiyentlarning qoniqarli tiklanishi olindi.
4. Identifikatsiya masalasi uchun parallel algoritm ishlab chiqildi. Bu hisoblash vaqtini 2,6 martagacha qisqartirish imkonini berdi.
5. Keyk qatlam hosil qilib suspenziya suzulishi (cake filtration) va muhitning ichki qismida cho‘kma hosil bo‘lib birjinslimas suyuqliklar sizishi (deep bed filtration) modellari kombinatsiyasidan iborat gibrid model ishlab chiqildi.
6. G‘ovak muhitda birjinslimas suyuqliklarning sizishi gibrid modeliga asoslanib, masalani sonli yechish uchun samarali algoritmlar ishlab chiqildi.
7. G‘ovak muhitda suspenziya sizishi gibrid modeli ko‘p bosqichli cho‘kma kinetikasini hisobga olgan holda takomillashtirildi. Ko‘p bosqichli cho‘kma kinetikasini qo‘shish turli intensivlik zonalari va “yuklanish” va "eskirish" hodisalarini ifodalovchi "sinish"lar bilan namoyon bo‘ladigan o‘ziga xos konsentratsiya profillari hosil bo‘lishi ko‘rsatildi.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREES DSc.03/2025.27.12.FM/T.09.03 UNDER SAMARKAND STATE UNIVERSITY NAMED AFTER SHARAF RASHIDOV

SAMARKAND STATE UNIVERSITY NAMED AFTER SHARAF RASHIDOV

FAYZIEV BEKZODJON MURTAZAEVICH

INVESTIGATION OF INHOMOGENEOUS FLUIDS FILTRATION PROCESSES IN POROUS MEDIA BASED ON MULTI-STAGE KINETICS

01.02.05 – Fluid and gas mechanics

DISSERTATION ABSTRACT OF THE DOCTOR OF SCIENCE (DSc) ON PHYSICAL AND MATHEMATICAL SCIENCES

Samarkand – 2026

The theme of dissertation of doctor of science (DSc) was registered at the Supreme Attestation Commission under Academy of Science of the Republic of Uzbekistan under number B2026.1.DSc/FM318.

The dissertation has been prepared at Samarkand State University named after Sharaf Rashidov.

The abstract of the dissertation is posted in three languages (uzbek, english, russian (resume)) on the website (www.samdu.uz) and the "Ziyonet" Information and educational portal (www.ziyonet.uz).

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Dissertation is possible to review in Information-resource center at Samarkand State University named after Sharaf Rashidov (is registered № 57) (Address: University str. 15, Samarkand, 140104, Uzbekistan, tel: (+99866) 239-11-40).

Abstract of dissertation sent out on «26» August 2026 year
(Mailing report № 1 on «26» August 2026 year)

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INTRODUCTION (abstract of PhD thesis)

Actuality and demand of the theme of dissertation. In the world, special attention is paid to the rational use of water resources, the protection of groundwater and the in-depth study of the processes of substance transport in porous media. According to the United Nations, “in 2024, 2.2 billion people still lacked safely managed drinking water”[5]. The deterioration of the quality of water resources, the spread of pollutants in porous media, the mineralization of groundwater and the increase in anthropogenic impacts create the need for accurate mathematical modeling of solute transport processes. In this regard, special attention is paid to improving models of the filtration of inhomogeneous fluids in porous media, taking into account many physical, mechanical and chemical factors.

Scientific research is being conducted in the world to ensure the effective development of oil and gas fields, predict the filtration of groundwater, assess the distribution of chemical reagents in porous media, and reduce environmental risks. In this direction, in particular, due to the emergence of problems of inhomogeneous fluid filtration in porous and fracture-porous media in many technological and production processes, research on modeling the processes of filtration of dispersed systems in porous media is considered a priority. At the same time, due to the global increase in demand for water resources and the increasing use of secondary methods associated with water injection in the process of oil and gas production, improving mathematical models of the processes of inhomogeneous fluid filtration and solute transport in porous media and creating effective algorithms to solve the problems are urgent tasks.

In our republic, within the framework of fundamental and applied research aimed at studying the processes of mass transport in porous media, research is being conducted on the introduction of modern mathematical modeling approaches, conducting computational experiments, and creating automated software tools based on visual analysis of the results. These studies serve to deepen the understanding of solute transport processes, improve models, and expand their scope of application.

In our republic, special attention is being paid to "... scientific and methodological support for the introduction of modern methods of managing the activities of the drinking water supply and wastewater services sector, as well as the introduction of innovative technological solutions and scientifically based modern approaches to the sector"[6]. In particular, this dissertation work contributes to the implementation of the goals and objectives set out in the regulatory and legal documents adopted in this area, which include the Decree of the President of the Republic of Uzbekistan No. PF-6024 dated July 10, 2020 "On Approval of the Concept of Water Management Development of the Republic of Uzbekistan for 2020 - 2030" and the Resolution No. PQ-4040 dated October 30, 2019 "On Additional Measures for the Development of Drinking Water Supply and Sewerage

[5] https://www.un.org/sustainabledevelopment/water-and-sanitation/

[6] Decree of the President of the Republic of Uzbekistan No. PF-6024 dated July 10, 2020 “On approval of the Concept of the Development of water sector of the Republic of Uzbekistan for 2020-2030”.

Systems in the Republic of Uzbekistan"[7], which set the introduction of scientifically based approaches to water resources management, the use of modern technologies and mathematical modeling methods as an important task. In implementing these tasks, it is important, in particular, to build new water treatment facilities and modernize existing ones based on economical and environmentally friendly materials, including determining the optimal parameters of filters used in water treatment facilities using mathematical modeling and modern information technologies.

This dissertation work to a certain extent serves the implementation of the goals and objectives stipulated in the Decrees of the President of the Republic of Uzbekistan No. PF-60 dated January 28, 2022 “On the Development Strategy of New Uzbekistan for 2022-2026”, No. PF-6024 dated July 10, 2020 “On Approval of the Concept of Water Management Development of the Republic of Uzbekistan for 2020-2030”, No. PQ-343 dated October 24, 2023 “On Additional Measures to Further Improve the Drinking Water Supply and Wastewater System”, and other regulatory legal documents related to this activity.

Dependence of research to priority directions of development of science and technologies of the republic. This study was performed in accordance with the Republic of Uzbekistan IV priority areas of science and technology. "Mathematics, Mechanics and Computer Science".

Review of foreign scientific research on the topic of the dissertation.[8]

Research on the processes of inhomogeneous fluid filtration in porous media is carried out at leading scientific centers and higher educational institutions of the world, including the American Petroleum Institute (API), the University of Texas, the University of Adelaide, the University of New South Wales (USA), the Institute of Petroleum Engineering (Germany), the Norwegian University of Science and Technology (Norway), the Institute of Oil and Gas (Scotland), the Delft University of Technology (Netherlands), the Oxford Institute of Energy Research (England), the China Petroleum University (China), the University of New South Wales (Australia), the PETRONAS Technological University (Malaysia), the Mexican Petroleum Institute (Mexico), the King Fahd University of Petroleum and Minerals (Saudi Arabia), the Institute of Petrochemistry of the Russian Federation SB, the Academician A. P. Krylov All-Russian Research Institute of Oil and Gas, the I. M. Gubkin Russian State University of Oil and Gas, the Tyumen State University of Oil and Gas, the Academician M.D. Millionshikov Grozny Oil Institute, Kazan Federal University, Kuban State Technical University, St. Petersburg State Mining Institute (Technical University), Ufa State Oil Technical University (Russia), Institute of Petrochemical Processes (NKZH), Oil and Gas Research Projects (Azerbaijan); Oil Industry Research Institute, Tashkent State Technical University, Samarkand State

[7] Resolution of the President of the Republic of Uzbekistan No. PQ-4040 dated November 30, 2018 “On additional measures to develop drinking water supply and sewage systems in the Republic of Uzbekistan”.

[8] A review of foreign scientific research on the topic of the dissertation was developed based on http://www.mathnet.ru; http://msp.org/jomms/about/cover/cover.html; http://link.springer.com; http://www.sciencedirect.com; http://www.dissercat.com/catalog/fiziko-matematicheskie-nauki and other sources.

University, Karshi State University, Tashkent Branch of the Gubkin Russian Oil and Gas Institute (Uzbekistan).

A number of scientific results have been obtained worldwide in the study of the processes of inhomogeneous fluid permeation in porous media, including the development of theoretical methods (Petroleum Engineering Institute, Germany, American Petroleum Institute, USA, Japan Petroleum Institute, Japan, Research Institute of Oil Exploration and Production, China, Institute of Oil and Gas, Russia, Institute of Petrochemistry of the SB RFA, Russia, All-Russian Oil and Gas Research Institute named after Academician A. P. Krylov, Russia, Institute of Oil and Gas Problems of the RFA, Russia, "UZLITINEFTGAZ", Uzbekistan).

Currently, research is being conducted in the world to develop a theory of secondary and tertiary treatment problems and find effective methods to solve them in order to purify drinking water and increase the oil yield of fields. In particular, research is being conducted to improve the theory of inhomogeneous fluid filtration in porous media and, on this basis, to develop effective technologies for purifying drinking water and oil production.

The degree of scrutiny of the problem. Research on the problems of inhomogeneous fluid filtration in porous media has been conducted by a number of scientists around the world: T. Iwasaki, D. M. Mins, K. Ives, M. Sharma, M. Elimelech, J. Bear, Ch. Tien, P. Bedrikovetsky, A. Adin, M. Rebhun, J. Herzig, V. Gitis, G. Barenblat, Yu. M. Shekhtman, A. Kh. Mirzadjanzade, Ye. V. Venetsianov, K. S. Basniyev, D. Mikhailov, A. Nikiforov and other foreign scientists.

In our republic, F.B.Abutaliyev, J.F.Fayzullayev, N.M.Mukhidinov, R.Sadullaev, I.Alimov, J.Akilov, B.Khuzhayorov, I.K.Khujaev, N.Ravshanov, Sh.Kayumov, V.F.Burnashev, J.Makhmudov and other scientists have made significant contributions to the development of mathematical models and calculation methods for the processes of filtration of liquids and gases in porous media. Mathematical modeling of the problems of substance transport and suspension filtration in porous media is considered one of the most important areas of modern science and technology, and the development of mathematical modeling of these processes opens up new opportunities for innovation and technological progress. The interaction between the fluid in motion and the walls of the porous medium during the filtration and mass transfer of inhomogeneous fluids in porous media is a complex process. When studying such problems, it is necessary to derive balance and kinetic equations taking into account phenomena such as convection, molecular diffusion, dispersion, and deposition, and analyze them together as a system of equations. This follows from an analysis of scientific works on the fluid filtration and solute transport in porous media, which shows that boundary value problems for nonlinear partial differential equations are mathematically complex and have not been sufficiently studied to date.

The connection of the dissertation topic with the research work of the higher educational institution, in which the dissertation is carried out. The dissertation research was carried out within the framework of the fundamental scientific research project of the Sharof Rashidov Samarkand State University OT-F4-64 “Development and numerical study of hydrodynamic models of the solute transport and the filtration of liquids in homogeneous porous media” (2017-2020) and FZ-2020092877 “Development and numerical study of mathematical models of the processes of anomalous solute transport and the filtration of liquids in heterogeneous porous media” (2022-2027).

The aim of the research is consists of developing mathematical models of the processes of solute transport and fluid filtration in porous media, performing numerical analysis, and identifying model parameters, taking into account various factors leading to complex kinetics.

Research tasks:

improvement of the mathematical model of the suspension filtration in porous media, taking into account convective transport, hydrodynamic dispersion, deposition of suspended particles and detachment in the backflow and numerical solution of the problems;

improvement of the mathematical model of the two-component suspension filtration in porous media, taking into account changes in porous medium characteristics, diffusion and multi-stage kinetics, and numerical analysis;

solution of inverse problems on the identification of parameters of mathematical models of the suspension filtration in porous media;

construction of a hybrid model of solute transport in porous media. In this case, the mathematical model of suspension filtration with the formation of a cake layer (cake filtration) is “stitched” with models of inhomogeneous fluid flow with the formation of deposition in the interior of the medium (deep bed filtration).

The object of the study is a porous medium and a heterogeneous liquid that forms a deposition in it.

The subject of the research is mathematical models, computational algorithms, and hydrodynamic analysis of the processes of suspension filtration in porous media.

Research methods. The dissertation used methods of mathematical modeling, continium mechanics, computational mathematics, and computational experimentation.

The scientific novelty of the research is as follows:

The model of filtration of suspensions in porous media has been improved based on the multi-stage kinetic equation, taking into account the variability of environmental parameters, "aging", and "charging" phenomena;

The mathematical model of the filtration process of two-component suspensions in porous media has been improved taking into account changes in porous medium characteristics, diffusion and multi-stage kinetics;

Algorithms for identifying the parameters of mathematical models of the process of solute transport in porous media have been developed;

An efficient algorithm based on parallel computing has been developed to solve inverse problems of solute transport in porous media;

A hybrid mathematical model has been developed that simultaneously takes into account the processes of cake filtration and deep bed filtration in porous media.

The hybrid model of solute transport in porous media has been improved based on the multi-stage kinetic equation, taking into account "aging" and "charging" phenomena.

Practical results of the research:

Improved mathematical models and computational algorithms for the process of solute transport in porous media have been used to solve problems in the mechanics of continious media and hydrogeology;

An effective algorithm for numerically solving the problems of filtration of single- and multi-component suspensions in porous media has been developed;

A software package has been created for numerical implementation and parameter identification of mathematical models of the process of solute transport in porous media.

The reliability of the results of the research. In the development of mass transfer models, fundamental conservation laws for multiphase systems were used, supplemented with phenomenological relations to ensure physical correctness. A phenomenological approach was used to develop a new kinetic equation for the deposition of dispersed particles in porous media and their re-entrainment to the fluid flow during the transfer process, and each assumption was carefully substantiated. When using numerical methods, their stability was checked and approximations of the required accuracy were used. The results obtained were analyzed in detail physically and their correspondence to real physical processes was assessed.

The scientific and practical significance of the research results. The scientific significance of the research results is explained by the fact that the theoretical basis and mathematical models, numerical methods and algorithms created taking into account many phenomena contribute to the theory of solute transport in porous media.

The practical significance of the obtained results serves to qualitatively and quantitatively assess the filtration processes in such areas as wastewater and drinking water treatment, secondary treatment of oil and gas fields, and hydrogeology.

Implementation of the research results. Based on the mathematical models, numerical algorithms and software tools developed for the development and numerical study of models of the solute transport in porous media based on multi-stage deposition kinetics:

The results of this dissertation were used in the implementation of the foreign project “Modified Homotopy Analysis Method for Solving System of Linear and Nonlinear Integral Equations and modelling on tension and scaling factor of the power cable” UMT/CRIM/2- 2/2/14 Vol. 4(44), Vote No 55233, carried out at the University of Malaysia Terengganu (UMT/FSKM/500-45 – reference number dated 16.01.2024). In particular, the project used numerical algorithms, based on finite difference schemes and developed software for solving systems of partial differential equations, have been used to numerically calculate the solution of the many problems in physics and engineering. Using the above algorithm, it was obtained the numerical solution of model problems for the equation reducible in hyperbolic systems. Using the software developed in dissertation made it possible to carry out computational experiments, which substantiates the convergence of the numerical solution to the exact solution of model problems and visualize the results;

Applied within the framework of the innovative project No. IL-21071166 “Creation of a vertical-axis wind turbine designed for low wind speeds” implemented at the Institute of Mechanics and Seismic Strength of Structures of the Academy of Sciences of the Republic of Uzbekistan (reference number 331-3, March 11, 2026). In particular, in the project, mathematical models were created using reaction-diffusion-type equations to model the stability of the wind flow at low speeds. The results of the methods and algorithms for solving inverse problems with coefficients for partial differential equations developed in the dissertation were used to find the parameters of these models. As a result, it was possible to determine the optimal parameters of a wind energy device with a vertical axis of rotation and moving working elements, which would operate effectively at low wind speeds;

The scientific results on the improved model of the filtration of two-component suspensions in porous media were used in the study of the problems of the filtration of inhomogeneous fluids in porous media in leading foreign journals (International Journal of Non-Linear Mechanics, Volume 167, December 2024, 104905, Annali di Matematica Pura ed Applicata, Volume 2012, May 2022, 2943–2964, Applied Mathematics and Mechanics, Volume 45, January 2024, 355–372). The application of scientific results made it possible to develop adequate models of the problems under consideration;

The scientific results on the algorithm for numerically solving the problem of suspension filtration in porous media were used in leading foreign journals (Journal of Applied Engineering Science, Volume 19, January 2021, 768, Geoenergy Science and Engineering,Volume 235, April 2024, 212696, Journal of Physics: Conference Series, Volume 1926, January 2021, 012001) to study problems corresponding to some nonlinear differential equations. The application of scientific results made it possible to obtain numerical solutions to the problems under consideration.

Approbation of the research results. The results of this research were discussed at 34 scientific and practical conferences, including 25 international and 9 republican scientific and practical conferences.

Publications of the research results. A total of 53 scientific works have been published on the topic of the dissertation, of which 13 articles were published in scientific publications recommended for publication of the main scientific results of doctoral dissertations by the Higher Attestation Commission of the Republic of Uzbekistan, including 10 in foreign and 3 in republican journals. All 10 articles published in foreign journals were published in journals indexed in the Scopus database, including 4 in journals falling into the Q1-Q2 quartile.

The structure and volume of the thesis. The thesis consists of an introduction, four chapters, conclusion and bibliography. The volume of the thesis is 165 pages.

MAIN CONTENTS OF THE DISSERTATION

The introduction substantiates the actuality and demand of the dissertation topic, describes a review of international scientific research on the topic, the purpose, objectives, object, and subject of the study, and its compatibility with the priority areas of scientific and technological development in the Republic of Uzbekistan. It also states the scientific novelty and practical results of the research. It emphasizes the reliability of the obtained results, their scientific and practical significance, information on the implementation of the research results, and approval of the work and published publications. The structure of dissertation is presented.

The first chapter of the dissertation provides a brief overview of the approaches used to model the processes of solute transport and suspension filtration in porous media, cake filtration and deep bed filtration. The general characteristics of the approaches used so far are identified. In particular, phenomenological approaches to model (matter balance and other) equations, as well as the generalized Darcy law, are used. Also, methods for numerically solving direct and inverse problems based on these models are reviewed.

In second section of the dissertation called "Mathematical modeling of suspension filtration in porous media with multistage deposition kinetics" given direct problems of suspension filtration in a porous medium with taking into account multistage deposition kinetics. Mathematical modeling of these processes requires taking into account a number of factors, including the hydrodynamic parameters of the flow, the structure of the porous medium, and the mechanisms of attachment and detachment of particles.

Let, from a certain moment in time $t>0$, a suspension with a concentration of suspended solids c_0, with a filtration velocity $v=const$ begins to flow into a homogeneous formation of length L, with initial porosity m_0, saturated with a clean (without suspended particles) fluid.

The mass balance equation of suspended particles in a suspension flow in the one-dimensional case has the form

$$m_0\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial x}+\frac{\partial \rho}{\partial t}=D\frac{\partial^2 c}{\partial x^2}, \tag{1}$$

where, ρ is the concentration of deposited particles, t is time, c is the particle concentration of the suspension, x is coordinate, D is diffusion coefficient.

The kinetics of deposition in which is taken in the form

$$\frac{\partial \rho}{\partial t}=\begin{cases}\beta_1 vc & for \quad 0<\rho\le\rho_1,\\ \beta_2 vc-\beta_3(1+\gamma|\nabla p|)\rho & for \quad \rho_1<\rho\le\rho_2,\\ \beta_2\frac{\rho_0}{\rho}vc-\beta_3(1+\gamma|\nabla p|)\rho & for \quad \rho_2<\rho<\rho_0,\\ 0 & for \quad \rho=\rho_0,\end{cases} \tag{2}$$

where, ρ_0 the total filter capacity, ρ_1 is the value of ρ at which "charging" is completed, β_1 is the coefficient associated with the effect of "charging", β_2 is the coefficient associated with the deposition of particles, β_3 is the coefficient associated with the release of particles, $|\nabla p|$ is the pressure gradient modulus, γ is constant coefficient.

For $|\nabla p|$ we use Darcy's law

$$v = K(m)|\nabla p|, \tag{3}$$

where $K(m)$ is filtration coefficient, $m = m_0 - \rho$ is current porosity.

For $K(m)$ we use Carmen-Kozeny equation

$$K(m) = k_0 m^3 / (1-m)^2,\ k_0 = const. \tag{4}$$

Initial and boundary conditions are in the following form

$$c(0,t) = c_0 = const,\ \frac{\partial c}{\partial x} = 0, \quad x = L, \quad t > 0,\ c(x,0) = 0, \quad \rho(x,0) = 0. \tag{5}$$

Problem (1) - (5) solved by using finite difference method. In the area $D = \{0 \le x \le L,\ 0 \le t \le T\}$ we introduce a grid. For this, we divide the interval $[0, L]$ with step h, and $[0,T]$ with step τ. As a result we have grid

$$\omega_{h\tau} = \left\{ (x_i, t_j),\ x_i = ih,\ i = 0,1,...I,\ t_j = j\tau,\ j = 0,1,...,J,\ \ \tau = T/J \right\}.$$

Instead of functions $c(t,x)$, $\rho(t,x)$ we consider the grid functions whose values at the nodes (x_i, t_j) respectively, denote by c_i^j, ρ_i^j.

The equations (1)-(4) is approximated on a grid $\omega_{h\tau}$ in the following form

$$m_0 \frac{c_i^{j+1} - c_i^j}{\tau} + v_0 \frac{c_i^{j+1} - c_{i-1}^{j+1}}{h} + \frac{\rho_i^{j+1} - \rho_i^j}{\tau} = D \frac{c_{i-1}^{j+1} - 2c_i^{j+1} + c_{i+1}^{j+1}}{h^2}, \tag{6}$$

$$\frac{\rho_i^{j+1} - \rho_i^j}{\tau} = \begin{cases} \beta_1 v c_i^j, & for \quad 0 < \rho_i^j \le \rho_1, \\ \beta_2 v c_i^j - \beta_3 \left(1 + \gamma |\nabla p|_i^{j+1}\right) \rho_i^j, & for \quad \rho_1 < \rho_i^j \le \rho_2, \\ \beta_2 \dfrac{\rho_0}{\rho_i^j} v c_i^j - \beta_3 \left(1 + \gamma |\nabla p|_i^{j+1}\right) \rho_i^j, & for \quad \rho_2 < \rho_i^j < \rho_0, \\ 0, & for \quad \rho_i^j = \rho_0, \end{cases} \tag{7}$$

$$|\nabla p|_i^{j+1} = \frac{v_0 \left(1 - m_0 + \left(\rho_{a,i}^j + \rho_{n,i}^j\right)\right)^2}{k_0 \left(m_0 - \left(\rho_{a,i}^j + \rho_{n,i}^j\right)\right)^3}. \tag{8}$$

The initial and boundary conditions (5) are also presented in the grid form

$$c_i^0 = 0\ ,\ \rho_i^0 = 0,\ i = \overline{0,I},\ c_0^j = c_0,\ j = \overline{0,J},\ c_I^j - c_{I-1}^j = 0,\ j = \overline{0,J}. \tag{9}$$

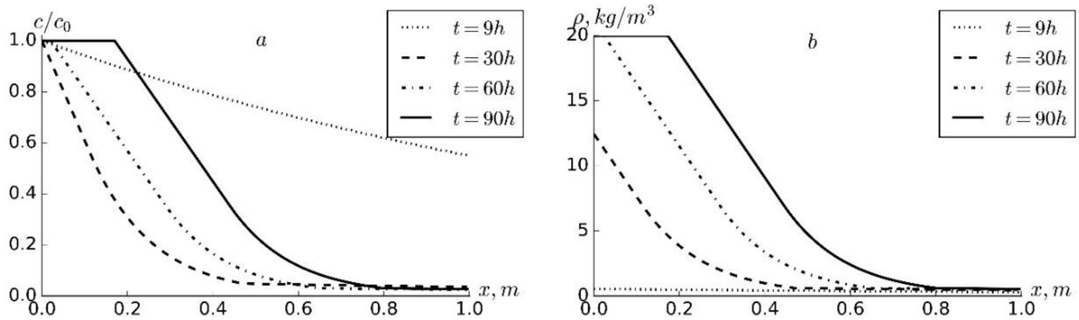

Fig. 1. Profiles of c / c_0 (*a*), ρ (*b*), at $\rho_1 = 0{,}6\,\text{kg/m}^3$, $\rho_2 = 7\,\text{kg/m}^3$.

Fig. 1 shows profiles of concentrations of suspended particles c/c_0 and deposited particles ρ at different time instants. At $t=9h$ (Fig. 1), advancement of the profile is observed, which corresponds to a period of almost complete absence of particle deposition. It can be explained that, during "charging" stage intensity of deposition is very low. With increasing current time, the process of particle deposition covers the entire filter zone, reaches its maximum capacity.

Mathematical model of two-component suspensions is a generalization of the same model for one component suspension. Particle mass balance equation is written for each fraction, which in the one-dimensional form is represented in the following form

$$m_0\frac{\partial c^{(i)}}{\partial t}+v\frac{\partial c^{(i)}}{\partial x}+\frac{\partial \rho^{(i)}}{\partial t}=D^{(i)}\frac{\partial^2 c^{(i)}}{\partial x^2}, \tag{10}$$

where m_0 is initial porosity, $c^{(i)}$ are volume concentrations of *i*-th suspension component solid particles (m^3/m^3), $\rho^{(i)}$ are deposition concentrations of *i*-th component (m^3/m^3), $D^{(i)}$ are diffusion coefficients for *i*-th suspension component (m^2/s), $i=1,2$ corresponds to the component numbers.

We denote the partial retention capacity with $\rho_0^{(i)}$. We can assume that the total capacity of the filter ρ_0 is determined by the first fraction, as well $\rho_0^{(1)}>\rho_0^{(2)}$. Consequently, the accumulation of deposition finishes when the total deposition reaches capacity. In addition, the deposition accumulation of the second fraction terminates when the partial capacity is reached, even if the total capacity is not exhausted, i.e. $\rho^{(1)}+\rho^{(2)}<\rho_0$. In addition, the "kinetic quality" of the first fraction is higher, i.e. $\beta_r^{(1)}/\beta_r^{(2)}>1,\ \beta_a^{(1)}/\beta_a^{(2)}>1,\ \beta_d^{(1)}/\beta_d^{(2)}>1$.

Kinetic equations we take in the following form

$$\frac{\partial \rho^{(1)}}{\partial t}=\begin{cases}\beta_r^{(1)}vc^{(1)}, & for \quad 0<\rho^{(1)}\le\rho_1^{(1)},\\ \dfrac{\beta_a^{(1)}vc^{(1)}}{1+\gamma^{(1)}\left|\nabla p\right|}-\beta_d^{(1)}\left(1+\omega^{(1)}\left|\nabla p\right|\right)\rho^{(1)}, & \text{for} \quad \rho_1^{(1)}<\rho^{(1)},\ \rho^{(1)}+\rho^{(2)}<\rho_0,\\ 0, & for \quad \rho^{(1)}+\rho^{(2)}=\rho_0\ ,\end{cases} \tag{11}$$

$$\frac{\partial \rho^{(2)}}{\partial t}=\begin{cases}\beta_r^{(2)}vc^{(2)}, & for \quad 0<\rho^{(2)}\le\rho_1^{(2)},\\ \dfrac{\beta_a^{(2)}vc^{(2)}}{1+\gamma^{(2)}\left|\nabla p\right|}-\beta_d^{(2)}\left(1+\omega^{(2)}\left|\nabla p\right|\right)\rho^{(2)}, & \text{for}\ \rho_1^{(2)}<\rho^{(2)}<\rho_0^{(2)},\rho^{(1)}+\rho^{(2)}<\rho_0,\\ 0, & for \quad \rho^{(2)}=\rho_0^{(2)}\ \text{or}\ \rho^{(1)}+\rho^{(2)}=\rho_0,\end{cases} \tag{12}$$

where $\gamma^{(i)}$, $\omega^{(i)}$ are constant coefficients.

The Darcy`s law in the one-dimensional case has the form

$$v=K(m)\left|\nabla p\right|,\ m=m_0-\left(\rho^{(1)}/\rho_d^{(1)}+\rho^{(2)}/\rho_d^{(2)}\right), \tag{13}$$

where $K(m)$ is filtration coefficient, $\rho_d^{(i)}$ are densities of particle materials, kg/m^3.

The initial and boundary conditions have the following form

$$c^{(i)}(x,0)=0,\ \ \rho^{(i)}(x,0)=0, \tag{14}$$

$$c^{(i)}(0,t)=c_0^{(i)}=\text{const},\ \frac{\partial c^{(i)}}{\partial x}=0,\quad x=L,\quad t>0. \tag{15}$$

The problem (10) - (15) is solved by the method of finite differences.

For the capacity of the passive zones, $\rho_0^{(1)}=15$ and $\rho_0^{(2)}=5$ were used. Near the inlet of the medium, over time, the concentration of the first component of the suspension becomes greater than the capacity of the media for the first component, i.e., $\rho^{(1)}>\rho_0^{(1)}$. Due to this fact, the capacity of the passive zone for the second component is saturated not only with concentration $\rho^{(2)}$, but also with $\rho^{(1)}$. Approximately at $t=30h$, the deposition concentration of the second component increases to some x and then decreases. Over time, the interval of increasing of $\rho^{(1)}$ expands.

Let we consider a semi-infinite homogeneous porous media with initial porosity m_0, filled with a homogeneous fluid. At the point $x=0$ starting from $t=0$ a two-component suspension with concentration $c_0=c_0^{(1)}+c_0^{(2)}$ ($c_0^{(1)}, c_0^{(2)}$ are concentrations of first and second type of particles in inlet suspension, respectively) is injected into the porous media with the velocity $v(t)=v=const$.

Next we consider a mathematical model for the filtration of two-component suspensions in a dual-zone porous medium with active and passive zones. In active zone, the deposition is formed in reversible form, whereas in passive zone it is formed in irreversible form. The balance equation with a constant flow velocity regime, including the diffusion term is

$$m\frac{\partial c^{(i)}}{\partial t}+v\frac{\partial c^{(i)}}{\partial x}+\frac{\partial \rho_a^{(i)}}{\partial t}+\frac{\partial \rho_p^{(i)}}{\partial t}=D^{(i)}\frac{\partial^2 c^{(i)}}{\partial x^2}, \tag{16}$$

where m is current porosity of media, $c^{(i)}$ is the concentration of the i^{th} component of the suspension (m^3/m^3), $\rho_a^{(i)}$, $\rho_p^{(i)}$ are respectively the concentration of deposition in the active and passive zones of the i^{th} component of the suspension (m^3/m^3), $D^{(i)}$ are diffusion coefficients for each component, (m^2/s), $i=1,2$ corresponds to the numbers of the components.

We generalized the kinetic equation of the process of attachment and detachment of particles in the active zone for a two-component suspension reflecting the dynamic factors as follows

$$\frac{\partial \rho_a^{(i)}}{\partial t}=\frac{\beta_a^{(i)}}{1+\gamma^{(i)}|\nabla p|}c^{(i)}-\beta_a^{(i)}\frac{\rho_a^{(i)}\left(1+\omega^{(i)}|\nabla p|\right)}{\rho_{a0}^{(i)}}c_0^{(i)}, \tag{17}$$

where $i=1,2.$, $\rho_{a0}^{(i)}$ are the capacities of the active zones for the *i*-th component of the suspension, $\beta_a^{(i)}$ is the coefficient characterizing the kinetics in the active zone of the i-th component of the suspension, $\gamma^{(i)}$, $\omega^{(i)}$ are constant coefficients characterizing intensity of influence of pressure gradient on particle attachment and detachment process in active zone.

The equations of kinetics of the formation of irreversible deposition in passive zone are as follows

$$\frac{\partial \rho_p^{(i)}}{\partial t}=\beta_p^{(i)}\phi_i\left(\rho_p^{(1)},\rho_p^{(2)}\right)c^{(i)}, \tag{18}$$

where $\beta_p^{(i)}$ are the coefficients characterizing the kinetics in the passive zone of the *i*-th component of the suspension.

The parameters $\phi_i\left(\rho_p^{(1)},\rho_p^{(2)}\right)$ characterizing the effect of compaction (aging) of the deposition are expressed, by the equations

$$\phi_1\left(\rho_p^{(1)},\rho_p^{(2)}\right)=\begin{cases}1 & \text{for} \quad 0<\rho_p^{(1)}\le\rho_{p1}^{(1)},\\ \rho_{p1}^{(1)}\big/\rho_p^{(1)} & \text{for} \quad \rho_{p1}^{(1)}<\rho_p^{(1)},\ \rho_p^{(1)}+\rho_p^{(2)}<\rho_{p0},\\ 0 & \text{for} \quad \rho_p^{(1)}+\rho_p^{(2)}=\rho_{p0},\end{cases} \tag{19}$$

$$\phi_2\left(\rho_p^{(1)},\rho_p^{(2)}\right)=\begin{cases}1 & \text{for} \quad 0<\rho_p^{(2)}\le\rho_{p1}^{(2)},\\ \rho_{p1}^{(2)}\big/\rho_p^{(2)} & \text{for} \quad \rho_{p1}^{(2)}<\rho_p^{(2)}<\rho_{p0}^{(2)},\ \rho_p^{(1)}+\rho_p^{(2)}<\rho_{p0},\\ 0 & \text{for} \quad \rho_p^{(2)}=\rho_{p0}^{(2)}\ or\,\rho_p^{(1)}+\rho_p^{(2)}=\rho_{p0},\end{cases} \tag{20}$$

where $\rho_{p1}^{(i)}$ are the limiting concentrations that in $\rho_p^{(i)}>\rho_{p1}^{(i)}$ begin compaction (aging) of the reversible form of the deposition for the corresponding component of the suspension.

The initial and boundary conditions are in the form

$$c^{(i)}(x,0)=0,\quad \rho_a^{(i)}(x,0)=\rho_p^{(i)}(x,0)=0,\ c^{(i)}(0,t)=c_0^{(i)}=\text{const}. \tag{21}$$

To solve the problem (16) - (21) we use finite difference method. Let us analyze the numerical results. In Fig. 2 profiles of changes $c^{(i)}$, $\rho_a^{(i)}$ and $\rho_p^{(i)},(i=1,2)$ are shown without dynamic factors and diffusion effects. Over time, the values of, $c^{(i)}$, $\rho_a^{(i)}$ and $\rho_p^{(i)}$ at fixed points in the reservoir increase (Fig. 2). For the capacity of the passive zones $\rho_{p0}^{(1)}=0{,}06$, $\rho_{p0}^{(2)}=0{,}03$ were used. It can be seen from Fig. 2c that near the inlet of the medium, over time, the concentration of the first component of the suspension becomes greater than the capacity of the passive zone for the first fraction, i.e. $\rho_p^{(1)} > \rho_{p0}^{(1)}$. Due to this fact, the capacity of the passive zone for the second component is saturated not only with concentration $\rho_p^{(2)}$, but also with $\rho_p^{(1)}$. Approximately at $t \approx 900$ *s,* the deposition concentration of the second component increases to some x and then decreases. Over time, the interval of increasing of $\rho_p^{(2)}$ expands.

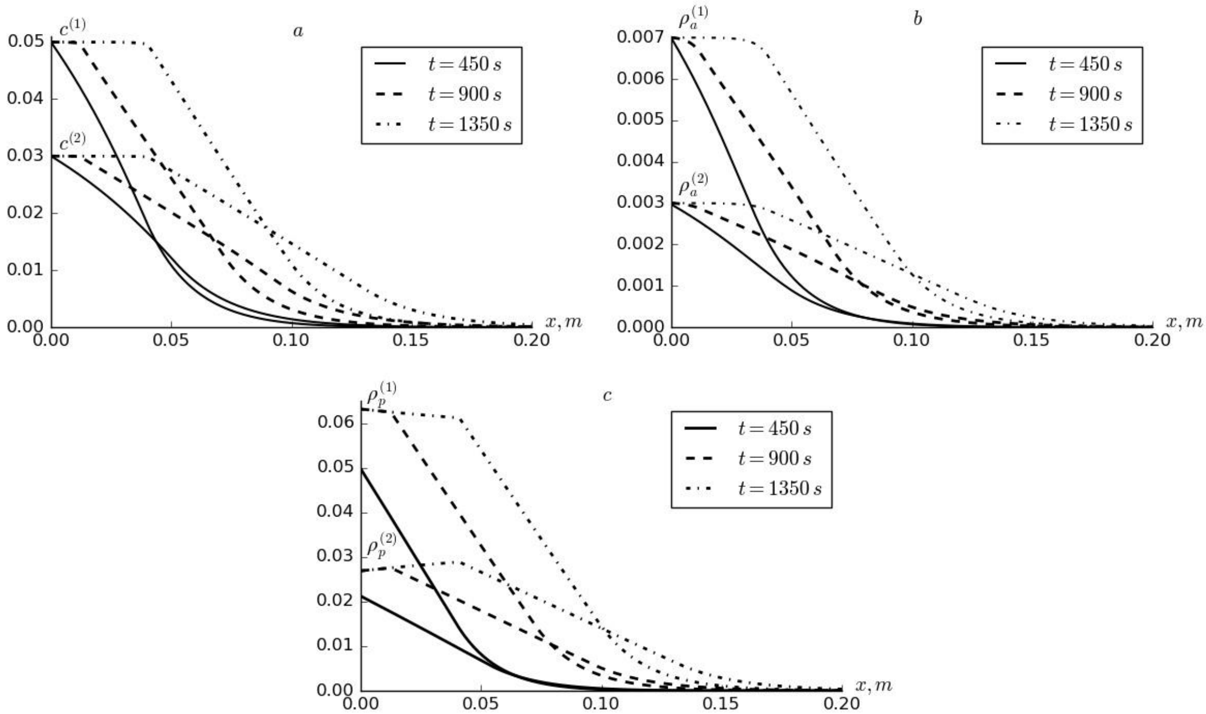

Fig. 2. Profiles of $c^{(1)}, c^{(2)}$(*a*), $\rho_a^{(1)}, \rho_a^{(2)}$(*b*), $\rho_p^{(1)}, \rho_p^{(2)}$ (*c*) at $\rho_{p1}^{(1)} = 0{,}015$, $\rho_{p1}^{(2)} = 0{,}006$, $\gamma^{(1)} = \gamma^{(2)} = \omega^{(1)} = \omega^{(2)} = 0$, $D^{(1)} = D^{(2)} = 0$.

When dynamic factors are not taken into account over time, the capacity of active zone for each component $\rho_{a_0}^{(1)}$ is completely saturated with deposition at the point *x=0.* In the case when dynamical factors are taken into account at the point *x=0* over time first $\rho_a^{(1)}$ increases, then decreases, and finally the dynamical equilibrium occurs before capacity of active zone is completely saturated with deposition.

Mathematical model of multi-component suspension filtration is a generalization of the single-component case, and the mass balance equations for each component of the suspension are ($i = 1, 2$)

$$\frac{\partial\left(mc^{(i)}\right)}{\partial t} + v\frac{\partial c^{(i)}}{\partial x} + \frac{\partial \rho_a^{(i)}}{\partial t} + \frac{\partial \rho_p^{(i)}}{\partial t} = D^{(i)}\frac{\partial^2 c^{(i)}}{\partial x^2}, \tag{22}$$

where m is the porosity of the medium, $c^{(i)}$ are concentration of the i^{th} component of suspension (m^3/m^3), $\rho_a^{(i)}$, $\rho_p^{(i)}$ are deposition concentrations of i^{th} component formed in the active and passive zones, respectively (m^3/m^3), $i=1,2$ correspond to the component numbers.

Kinetic equation for passive zone is same as above, but kinetic equation for active zone has been modified taking into account multistage deposition kinetics

$$\frac{\partial \rho_a^{(i)}}{\partial t}=\begin{cases}\beta_r^{(i)} v c^{(i)}, & 0<\rho_a^{(i)}\le \rho_{ar}^{(i)},\\ \beta_a^{(i)} v c^{(i)}-\beta_d^{(i)}\rho_a^{(i)}, & \rho_{ar}^{(i)}<\rho_a^{(i)}<\rho_{a0}^{(i)},\\ 0, & \rho_a^{(i)}=\rho_{a0}^{(i)},\end{cases} \tag{23}$$

where $\beta_r^{(i)}$ are the kinetic coefficients associated with the "charging" effect, $\beta_a^{(i)}$, $\beta_d^{(i)}$ are the coefficients representing the intensity of deposition of solid particles and their addition to the flow, respectively.

From Fig. 3b it can be said that the effect of the "charging" event in the active zone significantly affects the changing profiles of the concentration. As the parameter increases, two zones are formed: one with $\rho_a^{(i)}\le\rho_{ar}^{(i)}$ and the other $\rho_a^{(i)}>\rho_{ar}^{(i)}$. In these zones, the filteration characteristics have different rates of change (Fig. 3b). The formation of two zones is clearly visible in Figure 3. The rate of change of deposition concentration in the active zone also affects the concentration of suspended particles and deposition characteristics in the passive zone.

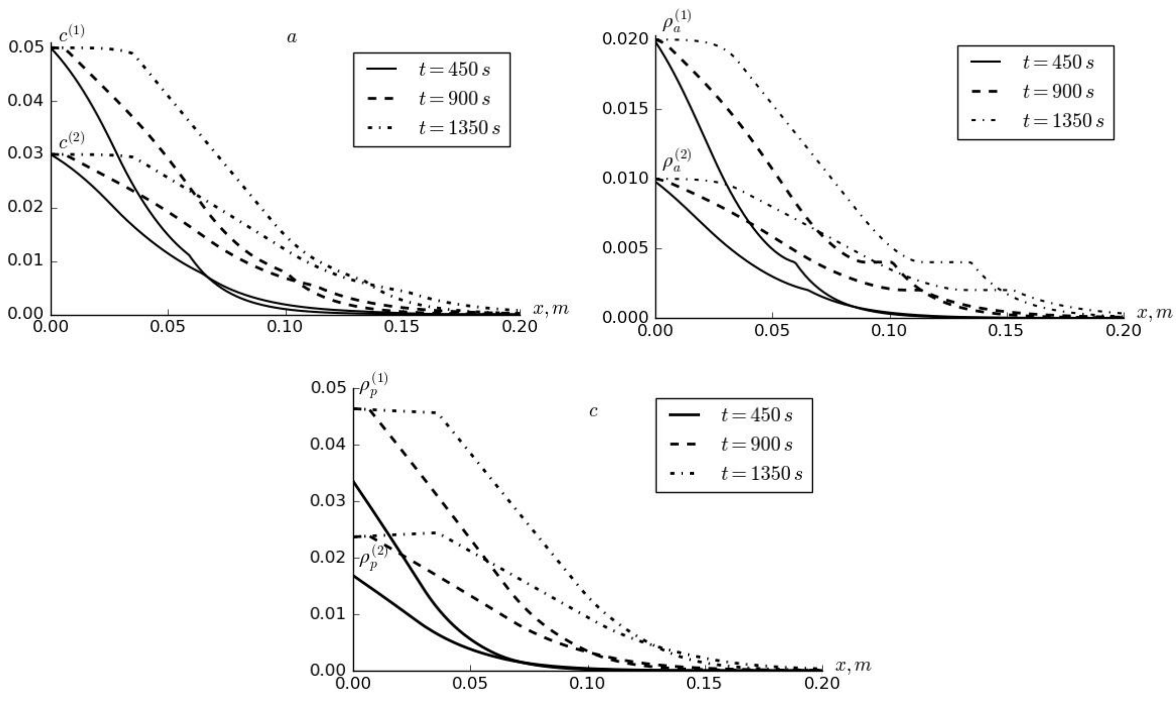

Fig. 3. Profiles of $c^{(1)}, c^{(2)}$ (a), $\rho_a^{(1)},\rho_a^{(2)}$ (b), $\rho_p^{(1)},\rho_p^{(2)}$ (c) at $\rho_{ar}^{(1)}=0{,}004$, $\rho_{ar}^{(2)}=0{,}002$.

In chapter 2 named “Inverse problems of filtration of suspension in porous media” inverse problems formulated and solved numerically to determine the model parameters.

Consider the following problem: into a homogeneous porous media of length L, with initial porosity m_0, saturated with a clear (particle-free) fluid, from a certain time $t>0$, a suspension with a concentration of suspended particles c_0 begins inflow with a filtration velocity $v(t)=v_0=\text{const}$. Here we use the model of suspension filtration in a porous medium with multistage deposition kinetics.

$$m_0\frac{\partial c}{\partial t}+v_0\frac{\partial c}{\partial x}+\frac{\partial \rho}{\partial t}=D\frac{\partial^2 c}{\partial x^2}, \tag{24}$$

$$\frac{\partial \rho}{\partial t}=\begin{cases}\beta_r v_0 c, & 0<\rho\le\rho_r,\\ \beta_a v_0 c-\beta_d\rho, & \rho_r<\rho\le\rho_0,\\ 0, & \rho=\rho_0,\end{cases} \tag{25}$$

where c is the suspension mass concentration (kg/m^3), ρ is the deposition mass concentration (kg/m^3), ρ_0 is the total capacity of the filter (kg/m^3), D is the diffusion (or dispersion) coefficient (kg/m^3), ρ_1 is the value ρ at which "charging" ends $(1/m), \beta_r$ is coefficient associated with the "charging" effect $(1/m), \beta_a$ is coefficient associated with particle attachment $(1/m)$, β_d is coefficient associated with particle detachment (1/s).

The initial and boundary conditions have the form

$$\begin{aligned}&c(0,t)=c_0=\text{const},\ \frac{\partial c}{\partial x}=0,\quad x=L,\quad t>0,\\ &c(x,0)=0,\quad \rho(x,0)=0.\end{aligned} \tag{26}$$

The problem (24) - (26) corresponds to the direct statement. With known coefficients in (24), (25), as well as c_0 in (26), the solutions $c(x,t)$, $\rho(x,\text{t})$ will be found. When solving the inverse problem, some coefficients in (24), (25) are unknown and must be determined. This requires additional information about the solution of the direct problem. To do this, we solve the direct problem with known values of the parameters. As additional information we use values of the suspended particles concentration at some given points $x_l\in(0,L),\quad l=\overline{1,n}$ which we denote as $Z_l(t)$.

Let us conduct a quasi-real experiment, for this we solve the direct problem (24)-(26) with known $\beta_r^{exact}=0.8$ 1/m, $\beta_a^{exact}=7.5$ 1/m. According to the results of numerical calculations, the grid function $z_l^j=z_l(t_j),\ j=0,1,...,M$ will be found. Calculations are also carried out in the case when the function $z(t)$ is perturbed using random inaccuracies in the following way:

$$\left(z_l^j\right)_\delta=z_l^j+2\delta\left(\sigma^j-\frac{1}{2}\right),$$

where σj is a random function uniformly distributed over the interval $[0;\ 1]$, δ is the error level. Perturbated function $z_l(t)$ we denote as $z_l^{\delta}(t)$.

Consider the inverse problem of suspension filtration, which consists of determining the coefficients β_a and β_r of the system of equations (24), (25) using additional information about the solution of the direct problem. To solve the problem, we use the identification method. We set the inverse problem as follows: determine the coefficients β_a, β_r from the minimum condition for the following functional

$$\Phi(\beta_r,\beta_a)=\sum_{l=1}^{n}\int_0^T\left[c(x_l,t)-z_l(t)\right]^2 dt, \tag{27}$$

The stationarity condition for the functional (27) has the form

$$\frac{d\Phi(\boldsymbol{\gamma})}{d\boldsymbol{\gamma}}=2\cdot\sum_{l=1}^{n}\int_0^T\left[c(x_l,t)-z_l(t)\right]\cdot\omega(z_l,t)dt=0, \tag{28}$$

where ω is a column vector, γ is a row vector,

$$\frac{d\rho}{d\boldsymbol{\gamma}}=\omega=(\omega_{11},\omega_{12})^T, \quad \boldsymbol{\gamma}=(\beta_r,\beta_a) \tag{29}$$

where $\omega_{11}=\dfrac{\partial c}{\partial\beta_r},\omega_{12}=\dfrac{\partial c}{\partial\beta_a}$ sensitivity function with respect to coefficient β_a and β_r respectively.

We expand the function *c* in a neighborhood of γ^s up to second-order terms

$$c^{s+1}(x,t)\approx c^s(x,t)+\left(\boldsymbol{\gamma}^{s+1}-\boldsymbol{\gamma}^s\right)\cdot\omega^s(x,t). \tag{30}$$

Substituting the expansion (30) into the relation (27) we obtain the following system of linear algebraic equations with respect to β_r^{s+1}, β_a^{s+1}

$$\begin{cases} a_{11}\beta_r^{s+1}+a_{12}\beta_a^{s+1}=b_1, \\ a_{21}\beta_r^{s+1}+a_{22}\beta_a^{s+1}=b_2, \end{cases} \tag{31}$$

where

$$a_{11}=\sum_{l=1}^{n}\int_0^T\omega_{11}^{s\,2}(x_l,t)dt,$$

$$a_{22}=\sum_{l=1}^{n}\int_0^T\omega_{12}^{s\,2}(x_l,t)dt,$$

$$a_{12}=a_{21}=\sum_{l=1}^{n}\int_0^T\omega_{11}^{s\,2}(x_l,t)\cdot\omega_{12}^{s\,2}(x_l,t)dt, \tag{32}$$

$$b_1=\sum_{l=1}^{n}\int_0^T\left[\omega_{11}^s(x_l,t)\beta_r^s+\omega_{12}^s(x_l,t)\beta_a^s-c^s(x_l,t)+z_l(t)\right]\omega_{11}^s(x_l,t)dt,$$

$$b_2=\sum_{l=1}^{n}\int_0^T\left[\omega_{11}^s\left(x_l,t\right)\beta_r^s+\omega_{12}^s\left(x_l,t\right)\beta_a^s-c^s\left(x_l,t\right)+z_l\left(t\right)\right]\omega_{12}^s\left(x_l,t\right)dt\ ,$$

We differentiate (24)-(26) with respect to β_r and get the following problem

$$m_0\frac{\partial\omega_{11}}{\partial t}+v_0\frac{\partial\omega_{11}}{\partial x}+\frac{\partial\omega_{21}}{\partial t}=D\frac{\partial^2\omega_{11}}{\partial x^2},\tag{33}$$

$$\frac{\partial\omega_{21}}{\partial t}=\begin{cases}v_0c+\beta_r v_0\omega_{11}, & 0<\rho\le\rho_r,\\ \beta_a v_0\omega_{11}-\beta_d\omega_{21}, & \rho_r<\rho\le\rho_0,\\ 0, & \rho=\rho_0,\end{cases}\tag{34}$$

$$\omega_{11}\left(x,0\right)=0,\ \ \omega_{21}\left(x,0\right)=0,\ \ \omega_{11}\left(0,\mathrm{t}\right)=0,\qquad \left.\frac{\partial\omega_{11}}{\partial x}\right|_{x=L}=0\,.\tag{35}$$

where $\omega_{22}=\dfrac{\partial\rho}{\partial\beta_a}$ sensitivity function with respect to coefficient β_r.

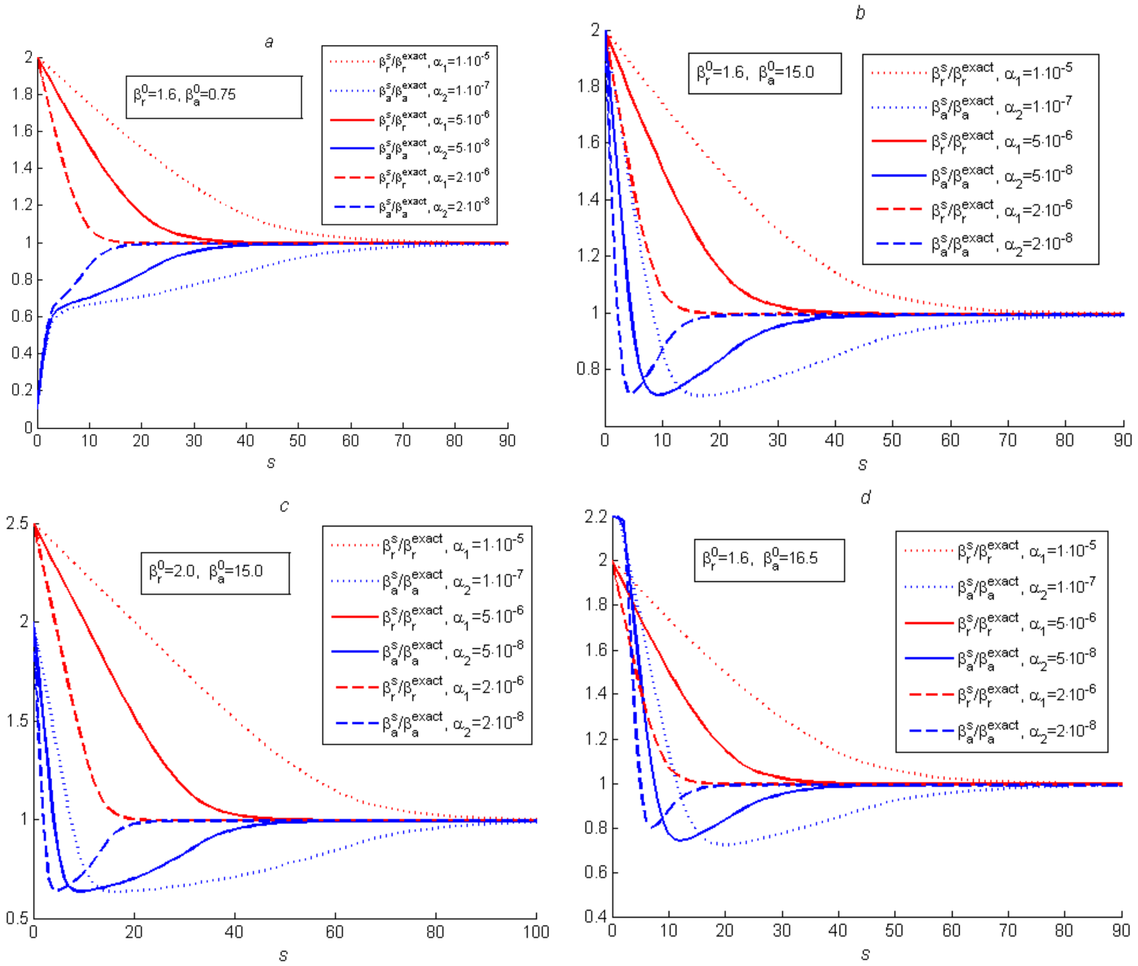

Fig. 4. Recovery of coefficients β_r, β_a with unperturbed initial data ($\delta=0$), values of β_r^{exact}, β_a^{exact} are as in the figure.

Differentiating (24)-(26) with respect to β_a and we get the following problem

$$m_0\frac{\partial\omega_{12}}{\partial t}+v_0\frac{\partial\omega_{12}}{\partial x}+\frac{\partial\omega_{22}}{\partial t}=D\frac{\partial^2\omega_{12}}{\partial x^2},\tag{36}$$

$$\frac{\partial \omega_{22}}{\partial t} = \begin{cases} \beta_r v_0 \omega_{12}, & 0 < \rho \le \rho_r, \\ v_0 c + \beta_a v_0 \omega_{12} - \beta_d \omega_{22}, & \rho_r < \rho \le \rho_0, \\ 0, & \rho = \rho_0, \end{cases} \qquad (37)$$

$$\omega_{12}(x,0) = 0, \quad \omega_{22}(x,0) = 0, \quad \omega_{12}(0,\mathrm{t}) = 0, \qquad \left.\frac{\partial \omega_{12}}{\partial x}\right|_{x=L} = 0, \qquad (38)$$

where $\omega_{22} = \frac{\partial \rho}{\partial \beta_r}$ sensitivity function with respect to coefficient β_a.

The algorithm for determining the coefficients β_r and β_a can be constructed as follows:

1. To set some initial approximations $\beta_r = \beta_r^0$ and $\beta_a = \beta_a^0$ (we set $s = 0$).

To solve problems (24)-(26), (33)-(35), (36)-(38), from $t = 0$ to $t = T$ and calculate the functions $c^s(x,t)$, $\rho^s(x,t)$, $\omega_{11}^s(x,t)$, $\omega_{21}^s(x,t)$, $\omega_{12}^s(x,t)$, $\omega_{22}^s(x,t)$

To solve the (31)-(32) system and obtain the following approximations β_r^{s+1}, β_a^{s+1} ;

To set $s = s+1$, $\beta_r = \beta_r^{s+1}$, $\beta_a = \beta_a^{s+1}$

5. To repeat steps 2,3,4 until the condition is satisfied

$$\left|\frac{\Phi^{s+1} - \Phi^s}{\Phi^s}\right| \le \varepsilon, \qquad \left|\frac{\beta_r^{s+1} - \beta_r^s}{\beta_r^s}\right| \le \varepsilon_1, \qquad \left|\frac{\beta_a^{s+1} - \beta_a^s}{\beta_a^s}\right| \le \varepsilon_2.$$

where ε, $\varepsilon_1, \varepsilon_2$ are sufficiently small numbers that determine the error of the problem solution.

Results of calculations to determine the coefficients β_r and β_a for various initial approximations β_r^0 and β_r^0 near the equilibrium point show that even near the equilibrium point, the approximation diverges (non-damped oscillations occur). Therefore, to solve the inverse problem, we use a modified first-order method. To do this, we replace the functional (27) of another form

$$\Phi(\gamma) = \sum_{l=1}^{n} \int_0^T \left[c(x_l,t) - z_l(t)\right]^2 dt + a\left(\gamma^{s+1} - \gamma^s\right), \qquad (39)$$

where $\gamma = (\beta_r, \beta_a)$ is a vector, $\alpha = (\alpha_1, \alpha_2)^T$ – column vector regularization parameter, components a_1 and a_2 are regularization parameters of coefficients β_r and β_a, respectively.

Fig.4 shows the results of calculations for the recovery of the coefficients β_r , β_a for various approximations β_r^0, β_a^0 depending on the regularization parameters α_1, α_2. The calculation results show that as the initial approximations β_r^0, β_a^0 move away from the equilibrium point, the number of iterations increases (Fig. 4 a◦, b, c, d). The number of iterations varies from twenty to one hundred, depending on the initial approximations β_r^0, β_a^0. Results show also (Fig. 4 a∘, b, c, d) that the number of iterations decreases with decreasing the regularization parameters α_1, α_2.

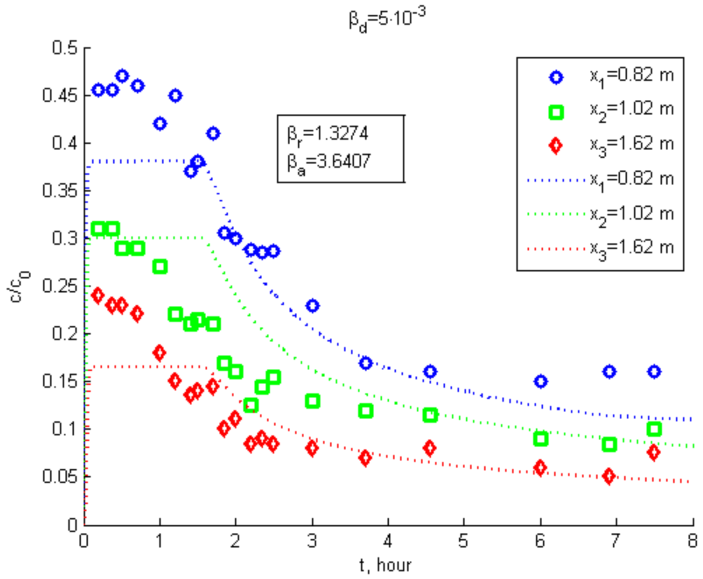

Fig. 5: Experimental data (dots) and model prediction (curves) for the experiments

Using the algorithm given above we also solve an inverse problem with initial data from laboratory experiments. Results are given in Fig.5. It can be seen that the results are satisfactory.

Let we consider a layer with length L and initial porosity m_0 filled with a homogeneous liquid. From $t > 0$ to $t \le t_1$ at the point $x = 0$ the inhomogeneous liquid containing suspended solid particles with concentration c_0 starts injecting to the layer with the constant filtration velocity U.

Suspension filtration occurs in porous media with active and passive zones, which means deposition formation, have reversible and irreversible forms, respectively. In this case, mathematical model of the process consists of mass balance equation and kinetic equations for both zones

$$\frac{\partial c}{\partial t} + U\frac{\partial c}{\partial x} + \frac{d_m}{m}\frac{\partial \rho_a}{\partial t} + \frac{d_m}{m}\frac{\partial \rho_p}{\partial t} = D_L\frac{\partial^2 c}{\partial x^2}, \tag{40}$$

$$\frac{d_m}{m}\frac{\partial \rho_a}{\partial t} = \beta_a c - \beta_d \frac{d_m}{m}\rho_a, \tag{41}$$

$$\frac{d_m}{m}\frac{\partial \rho_p}{\partial t} = \beta_p c, \tag{42}$$

where c is concentration of suspended particles in the fluid, d_m is the bulk density, m is the porosity, ρ_a and ρ_p are deposited particle concentration in active and passive zones, respectively, D_L is diffusion coefficient, β_a is reversible deposition rate coefficient, β_d is reversible deposition re-entrainment coefficient, β_p is irreversible deposition rate coefficient, x is the coordinate.

Initial and boundary conditions are following

$$c(x,0)=0,\quad \rho_a(x,0)=\rho_p(x,0)=0, c(0,t)=\begin{cases} c_0, & 0\le t\le t_1 \\ 0, & t>t_1 \end{cases}, \left.\frac{\partial c}{\partial t}\right|_{x=L}=0. \qquad (43)$$

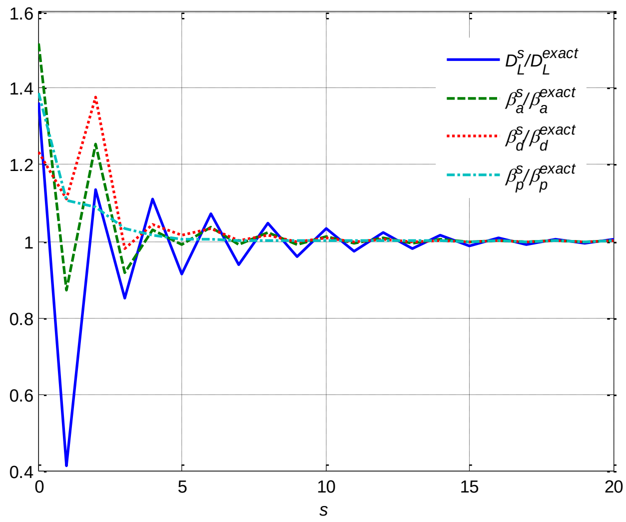

Fig. 6. *Recovery of coefficients* D_L, β_a, β_d, β_p around the equilibrium point with.

In this section we consider an inverse problem, which consists of determining the coefficients D_L, β_a, β_d, β_p in equations (40)-(43).

We determine these coefficients by minimizing the following functional

$$\Phi\left(D_L,\beta_a,\beta_d,\beta_p\right)=\int_0^T\left[c\left(L,t\right)-z\left(t\right)\right]^2 dt\ . \qquad (44)$$

Numerical calculations were carried out based on the above algorithm for solving the inverse problem. The results of calculations are shown in Fig.6. The calculation results show that all parameters have been restored with sufficient accuracy. It took up to 6-10 iterations to recover the coefficients with initial values near to the equilibrium point. The calculation results show that all parameters have been restored with sufficient accuracy. It took up to 8-20 iterations to recover the coefficients with initial values near to the equilibrium point.

We consider the possibilities of using parallel computing algorithms in determining the parameters of the suspension filtration model based on multi-stage kinetics in porous media. The main goal of parallel computing is to reduce the time to solve the problem. The task of parallel computing is to create a source of parallelism in the processes of solving problems in order to achieve the most efficient use of multiprocessor calculations (getting a parallel algorithm). Parallelism is an algorithm that allows to carry out tasks at the same time.

An experiment on computer also, carried out with developed parallel algorithm. We performed calculations only on four and eight core laptops. Using this algorithm allowed us to save time by 1.7 times when using 4-core laptops, and 2.6 times when using 8-core computers.

In chapter IV titled “Development of hybrid models of suspension filtration in porous media” deveoped a hybryd model. To study the combined process of filtration of suspensions with the formation of a cake layer on the filter surface (cake filtration) and filtration of heterogeneous liquids in porous media, where deposition of particles are formed inside the medium (deep bed filtration) we need adequate models to describe the transport and deposition of particles inside and on the surface of the filter medium. In this chapter we will present various models for the combined process of cake filtration and deep bed filtration, their numerical analysis are given. Mathematical models corresponding to these combined filtration processes, as already mentioned, will be called hybrid.

A suspension with a concentration of suspended solid particles C_{in} at a filtration velocity $u = const$ starts to enter a homogeneous filter of length s_0, with an initial porosity of φ_p, saturated with clean (particle-free) liquid at a certain point in time $t > 0$.

Growth factor of cake layer λ is introduced for separating the incoming flow of particles between filtration of suspensions to form a cake layer on the filter surface and filtration in a porous medium. Here λ is a dimensionless parameter ($0 \le \lambda \le 1$), such that λ part of the concentration of incoming particles immediately settles on the surface of the medium to form a cake layer, the remaining $1-\lambda$ part enters the medium and is subjected to filtration in the layers below.

The system of equations for cake filtration and deep bed filtration with a given velocity regime consists of the equation of balance and kinetics for each filtration zone, i.e. for the porous medium and the cake layer, as well as the equation of growth of the cake layer, which in the one-dimensional case is represented in the form

$$\frac{\partial C_p}{\partial t} + u\frac{\partial C_p}{\partial x} - D_p\frac{\partial^2 C_p}{\partial x^2} + \frac{\partial M_p}{\partial t} = 0, \quad x \in [0, s_0], \tag{45}$$

$$\frac{\partial M_p}{\partial t} = k_p u\left(C_p - \frac{(1-\lambda)\mathrm{C}_{in}}{M_{p0}}M_p\right), \quad x \in [0, s_0], \tag{46}$$

$$\frac{\partial C_c}{\partial t} + u\frac{\partial C_c}{\partial t} - D_c\frac{\partial^2 C_c}{\partial x^2} + \frac{\partial M_c}{\partial t} = 0, \quad x \in [s_0, \mathrm{s(t)}], \tag{47}$$

$$\frac{\partial M_c}{\partial t} = k_c u\left(C_c - \frac{(1-\lambda)\mathrm{C}_{in}}{M_{c0}}M_c\right), x \in [s_0, \mathrm{s(t)}], \tag{48}$$

$$\frac{ds}{dt} = \lambda\frac{C_{in}|u|}{(1-\varphi_c)}, \ s(0) = s_0, \ x = s(t), \tag{49}$$

$$\frac{dM_S}{dt} = \lambda|u|C_{in}, \qquad x = s(t), \tag{50}$$

where C_p, C_c are the concentrations of the suspension in the porous medium and cake layer, respectively, m^3/m^3, u is filtration velocity (m/s), D_p, D_c are the diffusion coefficients, m^2/s , M_p, M_c are concentration of deposition, m^3/m^3, M_s is the surface concentration, k_p, k_c are the kinetic coefficients expressing the intensity of deposit formation, M_{p0}, M_{c0} are the maximum value of the deposit formed, capacity, φ_c is the cake porosity, C_{in} is the concentration of solid particles at the inlet, and the indices p,c correspond to the porous medium and cake layer, respectively.

At the moving surface $x = s(t)$ (the top of the cake layer), a boundary condition is set for C_c. In the moving boundary model, since some λ part of the particles are immediately removed to the cake , only $1-\lambda$ the incoming concentration actually enters the cake as a flowing suspension. This was implemented by changing the inlet condition for C_c at the surface

$$uC_c - D_c \frac{\partial C_c}{\partial x} = (1-\lambda)uC_{in}, \quad x = s(t). \tag{51}$$

On the left border we use the following condition

$$\frac{\partial C_p}{\partial x} = 0, \qquad x = 0. \tag{52}$$

At the boundary of the porous medium with the cake we use the conditions

$$C_p = C_c, x = s_0, \tag{53}$$

$$uC_p - D_p \frac{\partial C_p}{\partial x} = uC_c - D_c \frac{\partial C_c}{\partial x}, \quad x = s_0. \tag{54}$$

Taking into account condition (53) for (54) we can write the following:

$$D_p \frac{\partial C_p}{\partial x} = D_c \frac{\partial C_c}{\partial x}. \tag{55}$$

We solve this system under the following initial conditions

$$C_p(0,x) = 0, \tag{56}$$

$$C_c(0,x) = 0. \tag{57}$$

Fig. 7 shows the concentration profiles C_c, C_p, M_c, M_p,. From Fig. 9a it is evident that the concentration of suspended particles in the cake layer changes with increasing deposit layer thickness *s*. It was noted that only $(1-\lambda)$ part of the incoming concentration C_{in} will be distributed over the cake layer. However, the concentration value could not reach its maximum value $C_c = (1-\lambda)C_{in}$, which is explained by condition (51).

Since $x = s(t)$ it is a moving boundary, at a fixed point in time the concentration C_c is higher at points close to this boundary, but over time, as part of this concentration passes into the porous medium, the concentration at fixed points decreases to a certain value (Fig. 7a). Fig. 7b shows the distribution of the suspended particles concentration in the porous medium, and it can be seen that at the boundary of the cake layer and the porous medium the value of the incoming concentration may be different. However, at other points of the filter the concentration gradually increases (Fig. 7b).

Figure 7c shows the concentration of solid particles in the cake layer and it can be seen that its value increases over time. The following conclusion can be made by comparing the change M_p in Fig. 7d with the change M_c in Fig. 7d: although the concentration value in the porous medium C_p around the point $x = s_0$ is less than C_c, it can be seen that the values M_p are greater than M_c at the given times. This is explained by the fact that the intensity of deposit formation in the porous medium is greater than in the cake layer.

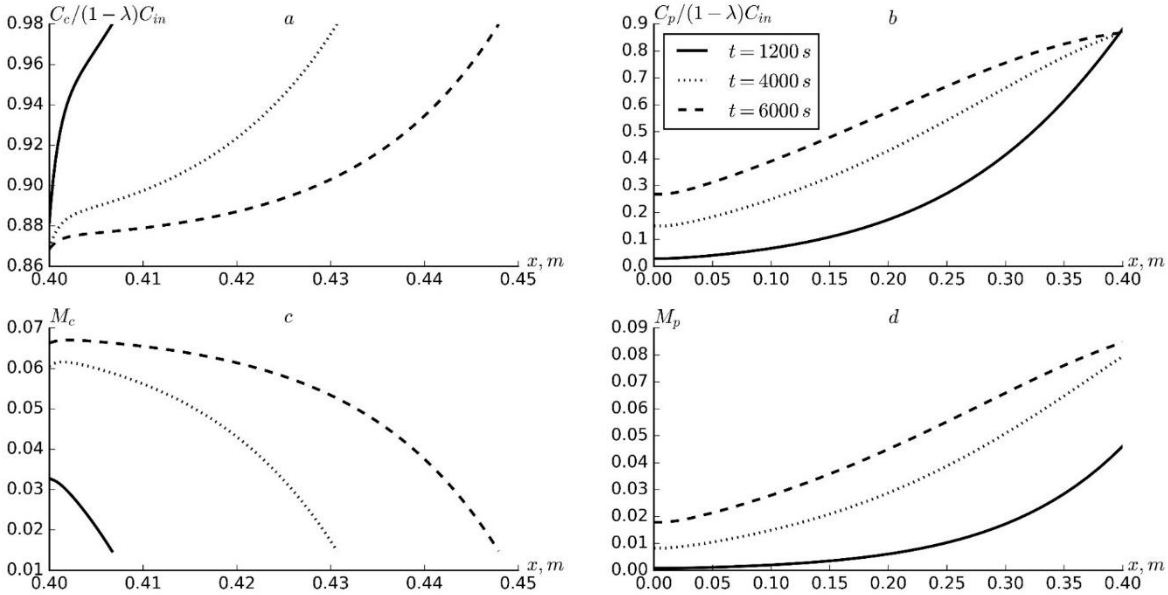

Fig.7. Profiles of $C_c(a)$, $C_p(b)$, $M_c(c)$, $M_p(d)$ at $D_p = 10^{-5}$ m²/s, $D_c = 10^{-6}$ m²/s .

The problem is generalized to the case of multistage deposition kinetics. The kinetics of deposition (46), (48) is taken in the form

$$\frac{\partial M_p}{\partial t} = \begin{cases} k_{rp} u C_p, & 0 \le M_p \le M_{p1}, \\ k_{ap} u C_p - k_{dp} M_p, & M_{p1} < M_p < M_{p0}, \\ 0, & M_p = M_{p0}, \end{cases} \tag{58}$$

$$\frac{\partial M_c}{\partial t} = \begin{cases} k_{rc} u C_c, & 0 \le M_c \le M_{c1}, \\ k_{ac} u C_c - k_{dc} M_c, & M_{c1} < M_c < M_{c0}, \\ 0, & M_c = M_{c0}. \end{cases} \tag{59}$$

It can be said that taking into account the multi-stage nature of the kinetic equations has a certain effect on the concentration profiles C_c, C_p, M_c, M_p. The smoothness of the graphs has decreased and kinks have appeared. In mulststage case the concentrations of deposition formed at fixed points at small time values are less than one stage case but for large time values the opposite picture can be observed. This is explained by the influence of the "charging" effect.

Considered finite filter with length s_0 and porosity m_2 at the initial moment of time . A homogeneous liquid containing two type of solid particles of different sizes with concentration c_0 ($c_0 = c_{10} + c_{20}$), starting from $t > 0$ begins to enter the filter. Large particles in the suspension settle on the surface of the porous medium and form a deposit layer (cake), while small particles enter the porous medium and participate in the process of deep filtration. It is also observed that small particles settle in the cake layer.

Filtration equations in the filter

$$m_2 \frac{\partial c_2}{\partial t} + v \frac{\partial c_2}{\partial x} + \frac{\partial \sigma_2}{\partial t} = D_2 \frac{\partial^2 c_2}{\partial x^2}, \qquad x \in [0, s_0], \tag{60}$$

$$\frac{\partial \sigma_2}{\partial t} = k_{2a} v c_2 - k_{2d} \sigma_2, \qquad x \in [0, s_0], \tag{61}$$

where c_2 is concentration of suspended particles in suspension , v is filtration rate, m_2 is porosity coefficient of the porous medium , σ_2 is the concentration of deposit formed in a porous medium, D_2 is diffusion coefficient in a porous medium , k_{2a} is coefficient characterizing the deposition of solid particles in a porous medium, k_{2d} is coefficient characterizing the outflow of solid particles to the reverse flow in a porous medium.

The equation characterizing the growth of the cake layer is written as follows:

$$\frac{ds}{dt} = \frac{c_1 v}{(1 - m_1)}, \quad x = s(t), \tag{62}$$

where c_1 is concentration of large particles in suspension, $s(t)$ is cake layer thickness, m_1 is cake layer porosity coefficient .

The equations of filtration in the cake layer are as follows:

$$m_1 \frac{\partial c_2}{\partial t} + v \frac{\partial c_2}{\partial t} + \frac{\partial \sigma_1}{\partial t} = D_1 \frac{\partial^2 c_2}{\partial x^2}, \qquad x \in [s_0, s(t)], \tag{63}$$

$$\frac{\partial \sigma_1}{\partial t} = k_{1a} v c_2 - k_{1d} \sigma_1, \quad x \in [s_0, s(t)], \tag{64}$$

where σ_1 is the concentration of fine particles that form a precipitate in the cake layer, D_1 is the diffusion coefficient in the cake layer, k_{1a} is coefficient characterizing the deposition of fine solid particles in the cake layer, k_{1d} is coefficient characterizing the outflow of solid particles back to the flow in the cake layer.

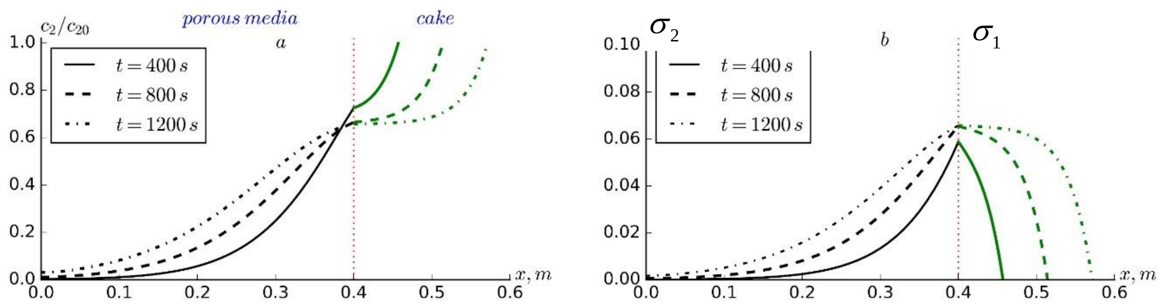

Fig. 8. Profiles of $c_2/c_{20}\,(a)$, $\sigma_1, \sigma_2\,(b)$.

Initial and boundary conditions appropriate to the problem were obtained and the problem was solved using the finite difference method.

Fig.8 shows the results obtained from the numerical solution of the hybrid mathematical model of filtration in a porous medium. The graphs show the concentrations of suspended particles in the cake layer and the porous medium, as well as the concentration of deposited particles, with the particle concentrations in the porous medium shown in black and in the cake layer in green.

Analysis of the results shows that the thickness of the cake layer increases over time, and the concentration of suspended particles and the concentration of deposit both in the cake layer and in the porous medium increase proportionally.

At fixed points, a decrease in the distribution of suspended particle concentration in the cake layer is observed over time due to the advance of the front. This property of the porous medium is preserved only at points close to point $s_0 = 0,4$, and when moving away from it, on the contrary, an increase in concentration over time can be observed. And in the deposit concentration, both in the porous medium and in the deposit layer, only an increase over time is observed.

CONCLUSIONS

The main results of the study are as follows:

1. A mathematical model has developed for a one-component suspension in a porous medium based on multistage deposition kinetics taking into all stages of deposit formation.
2. Mathematical model of two-component suspension filtration in porous media with the formation of two types of deposition is improved taking into account changes in porous medium characteristics and multistage deposition kinetics.
3. A coefficient inverse problem for identifying the parameters of a suspension filtration model in a porous medium is posed and numerically solved. To solve the problem, the identification method was first applied, which did not give satisfactory results. Therefore, a regularized first-order identification method was used. For sufficiently small values of the regularization parameter, obtained satisfactory recovery of the kinetic coefficients.
4. A parallel algorithm for the identification problem developed. It allowed to decrease the calculation time up to 2.6 times.
5. A hybrid model was developed, which is a combination of the cake filtration and deep bed filtration models.
6. Based on the hybrid model of inhomogeneous fluid filtration in porous media, effective algorithms were developed for numerical solution of the problem.
7. The hybrid model of suspension filtration in porous media was improved taking into account multi-stage sedimentation kinetics. It was shown that the inclusion of multi-stage sedimentation kinetics leads to the formation of specific concentration profiles, which are manifested by zones of different intensity and "kicks" representing the phenomena of "charging" and "aging".

НАУЧНЫЙ СОВЕТ DSc.03/2025.27.12.FM/T.09.03
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ
САМАРКАНДСКОМ ГОСУДАРСТВЕННОМ УНИВЕРСИТЕТЕ
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ИМЕНИ ШАРАФА РАШИДОВА

ФАЙЗИЕВ БЕКЗОДЖОН МУРТАЗАЕВИЧ

ИССЛЕДОВАНИЕ ПРОЦЕССОВ ФИЛЬТРАЦИИ НЕОДНОРОДНЫХ ЖИДКОСТЕЙ В ПОРИСТЫХ СРЕДАХ НА ОСНОВЕ МНОГОСТАДИЙНОЙ КИНЕТИКИ

01.02.05 – Механика жидкости и газа

АВТОРЕФЕРАТ ДИССЕРТАЦИИ
ДОКТОРА НАУК (DSc) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

Самарканд – 2026

Тема диссертации доктора наук (DSc) зарегистрирована в Высшей аттестационной комиссии при Академии Наук Республики Узбекистан за № B2026.1.DSc/FM318.

Диссертация выполнена в Самаркандском государственном университета имени Шарафа Рашидова.

Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице научного совета (www.samdu.uz) и на Информационно-образовательном портале «ZiyoNet» (www.ziyonet.uz).

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Защита диссертации состоится «11» сентября 2026 года в 10:00 часов на заседании Научного совета DSc.03/2025.27.12.FM/T.09.03 при Самаркандском государственном университета имени Шарафа Рашидова (Адрес: 104000, г. Самарканд, Университетский бульвар, 15. Тел.: (+99866) 239-11-40, факс: (+99866) 239-11-40, 239-12-47, e-mail: devonxona@samdu.uz).

С диссертацией можно ознакомиться в Информационно-ресурсном центре Самаркандского государственного университета имени Шарафа Рашидова (регистрационный номер 57). Адрес: 140104, Самарканд, Университетский бульвар, 15. (+99866) 239-17-25.

Автореферат диссертации разослан «26» августа 2026 года.
(реестр протокола рассылки № 1 от «26» августа 2026 года.)

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ВВЕДЕНИЕ (аннотация диссертации доктора наук (DSc))

Целью исследования является Целью исследования является разработка математических моделей процессов переноса веществ и фильтрации жидкости в пористых средах, проведение численного анализа и определение параметров модели с учетом различных факторов, приводящих к сложной кинетике.

Объект исследования является пористая среда и гетерогенная жидкость, образующая в ней осадок.

Предметом исследования являются математические модели, вычислительные алгоритмы и гидродинамический анализ процессов суспензионной фильтрации в пористых средах.

Научная новизна диссертационного исследования заключается в следующем:

Усовершенствована модель фильтрации суспензий в пористых средах на основе многостадийного кинетического уравнения;

Усовершенствована математическая модель процесса фильтрации двухкомпонентных суспензий в пористых средах с учетом изменений характеристик пористой среды, диффузии и многостадийной кинетики;

Разработаны алгоритмы идентификации параметров математических моделей процесса переноса растворенных веществ в пористых средах;

Разработан эффективный алгоритм, основанный на параллельных вычислениях, для решения обратных задач переноса веществ в пористых средах;

Разработана новая гибридная модель переноса веществ в пористых средах.

Усовершенствована гибридная модель переноса веществ в пористых средах на основе многостадийного кинетического уравнения.

Внедрение результатов исследования.

На основе математических моделей и численных алгоритмов, разработанных для разработки и численного исследования моделей переноса веществ в пористых средах на основе многостадийной кинетики осаждения:

Результаты данной диссертации были использованы при реализации зарубежного проекта «Модифицированный метод гомотопического анализа для решения систем линейных и нелинейных интегральных уравнений и моделирования натяжения и масштабного коэффициента силового кабеля» UMT/CRIM/2- 2/2/14 Том 4(44), номер работы 55233, выполненного в Университете Малайзии Теренггану (UMT/FSKM/500-45 – номер работы от 16.01.2024). В частности, в проекте использовались численные алгоритмы, основанные на схемах конечных разностей, и разработанное программное обеспечение для решения систем дифференциальных уравнений в частных производных, которые применялись для численного расчета решения многих задач в физике и технике. С помощью вышеуказанного алгоритма было получено численное решение модельных задач для уравнения, сводимого к гиперболическим системам. Использование программного обеспечения, разработанного в диссертации, позволило провести вычислительные эксперименты, подтверждающие сходимость численного решения к точному решению модельных задач и визуализировать результаты;

Применено в рамках инновационного проекта № ИЛ-21071166 «Создание ветротурбины с вертикальной осью вращения, предназначенной для низких скоростей ветра», реализуемого в Институте механики и сейсмостойкости конструкций Академии наук Республики Узбекистан (номер заявки 331-3, 11 марта 2026 г.). В частности, в проекте были созданы математические модели с использованием уравнений реакционно-диффузионного типа для моделирования устойчивости ветрового потока на низких скоростях. Результаты разработанных в диссертации методов и алгоритмов решения обратных задач с коэффициентами для дифференциальных уравнений в частных производных были использованы для нахождения параметров этих моделей. В результате удалось определить оптимальные параметры ветроэнергетического устройства с вертикальной осью вращения и подвижными рабочими элементами, эффективно работающего при низких скоростях ветра;

Научные результаты по усовершенствованной модели фильтрации двухкомпонентных суспензий в пористых средах были использованы при изучении проблем фильтрации неоднородных жидкостей в пористых средах в ведущих зарубежных журналах (International Journal of Non-Linear Mechanics, Volume 167, December 2024, 104905, Annali di Matematica Pura ed Applicata, Volume 2012, May 2022, 2943–2964, Applied Mathematics and Mechanics, Volume 45, January 2024, 355–372). Применение научных результатов позволило разработать адекватные модели рассматриваемых проблем;

Научные результаты по алгоритму численного решения задачи фильтрации суспензий в пористых средах были использованы в ведущих зарубежных журналах (Journal of Applied Engineering Science, Volume 19, January 2021, 768, Geoenergy Science and Engineering, Volume 235, April 2024, 212696, Journal of Physics: Conference Series, Volume 1926, January 2021, 012001) для изучения задач, соответствующих некоторым нелинейным дифференциальным уравнениям. Применение научных результатов позволило получить численные решения рассматриваемых задач.

Публикации результатов исследования. По теме диссертации опубликовано 53 научных работы, из которых 13 статей опубликованы в научных изданиях, рекомендованных Высшей аттестационной комиссией Республики Узбекистан для публикации основных научных результатов докторских диссертаций, в том числе 10 в зарубежных и 3 в республиканских журналах. Все 10 статей, опубликованных в зарубежных журналах, были опубликованы в журналах, индексируемых в базе данных Scopus, в том числе 4 в журналах, входящих в квартиль Q1-Q2.

Объем и структура работы. Диссертация состоит из введения, четырех глав, заключения и библиографии. Объем диссертации составляет 165 страниц.

E’LON QILINGAN ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo‘lim (part 1; 1 часть)

1. Khuzhayorov B., Fayziev B., Ibragimov G., Md Arifin N. A deep bed filtration model of two-component suspension in dual-zone porous medium // Applied Sciences (Switzerland), 2020. 10(8), 2793. https://doi.org/10.3390/app10082793 (№ 3 Scopus (Q1), IF=6,1, № 1 Web of Science (Q2), IF=2.7).
2. Fayziev B., Ibragimov G., Khuzhayorov B., Alias I.A. Numerical study of suspension filtration model in porous medium with modified deposition kinetics // Symmetry, 2020. 12(5), 696. https://doi.org/10.3390/sym12050696 (№ 3 Scopus (Q1), IF=5,2, № 1 Web of Science (Q2), IF=2.1).
3. Khuzhayorov B., Mustofoqulov J., Ibragimov G., Ali F.M., Fayziev B. Solute transport in the element of fractured porous medium with an inhomogeneous porous block // Symmetry, 2020. 12(6),1028. https://doi.org/10.3390/sym12061028 (№ 3 Scopus (Q1), IF=5,2, № 1 Web of Science (Q2), IF=2.1).
4. Fayziev B. A phenomenological model of suspension filtration in porous medium // International Journal of Applied Mathematics, Volume 33 No. 3, 2020, 511-521. http://doi.org/10.12732/ijam.v33i3.10 (№ 3 Scopus (Q3), IF=1.1).
5. Khuzhayorov B., Begmatov T., Kholiyarov E., Fayziev B. The inverse problem of determining the kinetic coefficients in the model of suspension filtration in a porous medium // Eurasian journal of mathematical and computer applications, Volume 10, Issue 4 (2022) 96 –106. https://ejmca.enu.kz/archive/ (№ 3 Scopus (Q3), № 1 Web of Science, IF=0.5).
6. Fayziev B., Makhmudov J., Mustofoqulov J., Begmatov T., Safarov R. Numerical Investigation of the Two-Component Suspension Filtration in a Porous Medium Taking into Account Changes in the Characteristics of the Porous Medium // WSEAS Transactions On Fluid Mechanics. vol. 18, pp. 214-220, 2023 https://doi.org/10.37394/232013.2023.18.20 (№ 3 Scopus (Q3), IF=1.4).
7. Fayziyev B. M. Improvement and numerical analysis of two-component suspension filtration model in porous media // Samarqand universiteti ilmiy axborotnomasi 2023 yil 5-son (141), 87-92. https://axborotnoma.uz/uz/research/2636/ (01.00.00; №2).
8. Fayziyev B.M., Hasim R.M., Begmatov T., Kholiyarov E. Identification of parameters in the model of suspension filtration in a porous medium // Samarqand universiteti ilmiy axborotnomasi 2023 yil 5-son (141), 132-139. https://axborotnoma.uz/uz/research/2640/ (01.00.00; №2).

9. Khuzhayorov B. Kh., Fayziev B. M., Kholiyarov E. Ch. Parallel algorithm for identification of parameters in the model of suspension filtration in a porous medium // Проблемы вычислительной и прикладной математики, 2024, 1(55): 81-97. https://journals.airi.uz/index.php/pvpm/article/view/5 (01.00.00; №9).
10. Fayziev B., Mustofoqulov J., Begmatov T. The Suspension Filtration Model in a Dual-Zone Porous Medium taking into Account the "Charging" Effect // AIP Conference Proceedings 3045, 050005 (2024), https://doi.org/10.1063/5.0197287 (№ 3 Scopus, IF=0.7).
11. B. Fayziev, O. Sagdullaev, S. Djalilov, O. Khaydarov, J.Mustofoqulov, E.Akhmedov, A. Mustafakulov, A.Toyirov, Degradable Solute Transport in Porous Media with Variable Hydrodynamic Dispersion // Applied Mechanics, 2025. 6(4), 89. https://doi.org/10.3390/applmech6040089 (№ 3 Scopus (Q2), IF=3,5, № 1 Web of Science (Q3), IF=1.7).
12. B.Fayziev; J. Makhmudov; F. Sulaymonov; T. Begmatov; M. D. Ghazi Study of the transfer equation using some difference schemes // AIP Conference Proceedings 3377, 020031 (2025), https://doi.org//10.1063/5.0300041, (№ 3 Scopus, IF=0.7).
13. B.Fayziev; F.Sulaymonov; T. Begmatov; M. R. Ilias Inverse problem of suspension filtration in a porous medium with modified deposition kinetics // AIP Conference Proceedings 3356, 020001 (2025), https://doi.org//10.1063/10.1063/5.0296877, (№ 3 Scopus, IF=0.7).

II boʻlim (part 2; 2 часть)

14. Khuzhayorov B., Fayziev B. Numerical analysis of two-component deep bed filtration model // Abstracts of the international scientific conference "Modern problems of applied mathematics and information technologies – Al-Khorezmiy 2018". (September 13–15, 2018) – Tashkent, NUUz named after Mirzo Ulugbek, 2018. P. 97.
15. Khuzhayorov B., Fayziev B. A deep bed filtration model of two-component suspension // Abstracts of the international scientific conference "Modern problems of applied mathematics and information technologies – Al-Khorezmiy 2018". (September 13–15, 2018) – Tashkent, NUUz named after Mirzo Ulugbek, 2018. Pp. 74-76.
16. Fayziev B.M., Begmatov T.I. A two-component deep bed filtration model with multistage deposition kinetics // Science – Technology – Education – Mathematics –Medicine. Abstracts of the Joint International Conference (May 16–17, 2019, Tashkent). P.49.
17. Fayziev B., Begmatov T., Yusupov N. A model of two-component suspension filtration in porous media taking into account “charching” effect // Absracts of the International conference “Actual problems of applied mathematics and information technology" Tashkent - Uzbekistan, November 14–15, 2019. p.57.

18. Khuzhayorov B., Fayziev B., Begmatov T. A model of suspension filtration in porous media with two deposition zones// Absracts of the International conference “Actual problems of applied mathematics and information technology" Tashkent - Uzbekistan, November 14–15, 2019. p.58.
19. Fayziev B., Yusupov N., Zokirov M. A mathematical model of two-component suspension in dual-zone porous medium “Innovatsion va zamonaviy axborot texnologiyalarini ta’lim, fan va boshqaruv sohalarida qo‘llash istiqbollari” mavzusidagi xalqaro ilmiy – amaliy onlayn konferensiyasi materiallari. 2020 yil 14-15 may –Samarqand, SamDU.
20. Fayziyev B.M., Begmatov T.I., Yusupov N. A Mathematical model of Two-component Suspension filtration in Porous medium with Dynamical Factors // Abstracts of communications of the Conference “Modern stochastic models and problems of actuarial mathematics”. – Karshi, Uzbekistan, September 25, 2020. Pp.14-15. https://www.researchgate.net/publication/344777400_Modern_StochAstic_Models_ProbleMs_Of_Actuarial_MaTHematics_MAMMOTH_Conference_-_Book_of_Abstracts_2020
21. Файзиев Б.М., Бегматов Т.И., Адилов А. Задача фильтрации суспензии с пористой среде с учетом динамических факторов // Abstracts of communications of the Conference “Modern stochastic models and problems of actuarial mathematics”. – Karshi, Uzbekistan, September 25, 2020. P.127-128. https://www.researchgate.net/publication/344777400_Modern_StochAstic_Models_ProbleMs_Of_Actuarial_MaTHematics_MAMMOTH_Conference_-_Book_of_Abstracts_2020
22. Khuzhayorov B., Fayziev B.M., Begmatov T.I. A mathematical model of two-component suspension filtration in a porous medium with “charging” effect // Abstracts of the International Online Conference Frontier in mathematics and computer science, October 12–15, 2020, National University of Uzbekistan 2020, Pp.80.
23. Файзиев Б.М., Бегматов Т.И, Эштемиров Б. Математическая модель фильтрации суспензии с многоступенчатой кинетикой осаждения в двухзонной пористой среде // «Амалий математика ва ахборот технологияларининг замонавий муаммолари» мавзусида халқаро илмий-амалий конференция материаллари, Бухоро, 2021.93-95 б.
24. Файзиев Б.М., Бегматов Т.И, Эштемиров Б. Обратная задача фильтрации суспензии в пористой среде с модифицированной кинетикой осадкообразования// « Ахборот коммуникация технологиялари ва дастурий таъминот яратишда инновацион ғоялар » мавзусида илмий-техник анжумани, ТАТУ СФ, 2021.36-39 б.
25. Файзиев Б.М., Бегматов Т.И., Усмонов А.И. Икки компонентли суспензияни фильтрлаб тозалашни ҳисоблаш учун дастур. № DGU 12979, Ўзбекистон Республикаси, Интелектуал мулк агентлиги. Тошкент, 2021.

26. Хужаёров Б.Х., Файзиев Б.М., Бегматов Т.И. Динамик омилларни ҳисобга олган ҳолда суспензия сизиши моделларини сонли таҳлил қилиш дастури. № DGU 12977, Ўзбекистон Республикаси, Интелектуал мулк агентлиги. Тошкент, 2021.
27. Fayziyev B., Nug'ayev S. Ikki zonali g`ovak muhitda ikki komponentali suspenziya sizishi jarayonining matematik modeli. // “Zamonaviy axborot, kommunikatsiya texnologiyalari va ta’lim tatbiqi muammolari” mavzusidagi respublika ilmiy-amaliy anjumani ma’ruzalar to‘plami, Samarqand, 9 aprel 2022-yil, 7-9 b.
28. Файзиев Б.М., Бегматов Т.И., Санаев М.Э. Обратная задача по определению кинетического коэффициентав модели фильтрации суспензии впористой среде // // “Zamonaviy axborot, kommunikatsiya texnologiyalari va ta’lim tatbiqi muammolari” mavzusidagi respublika ilmiy-amaliy anjumani ma’ruzalar to‘plami, Samarqand, 9 aprel 2022-yil, 11-13 b.
29. Fayziev B., Nugaev S., Sagdullaev O. A model of two-component suspension filtration in porous media taking into account multistage deposition kinetics. // “Amaliy matematika va axborot texnologiyalarining zamonaviy muammolari»” mavzusidagi xalqaro miqyosidagi ilmiy-amaliy anjuman materiallari. 2022 yil, 11-12 may, Бухоро – 2022, 254-255 б.
30. Файзиев Б.М., Бегматов Т.И., Санаев М.Э. Идентификация коэффициента кинетики в модели фильтрации суспензии в пористой среде. // “Amaliy matematika va axborot texnologiyalarining zamonaviy muammolari»” mavzusidagi xalqaro miqyosidagi ilmiy-amaliy anjuman materiallari. 2022 yil, 11-12 may, Бухоро – 2022, 360-361.
31. Бегматов Т.И., Файзиев Б.М., Санаев М.Э. Идентификация параметров модели процесса фильтрации в пористой среде с модифицированной кинетикой осадкообразования. // Современные научные исследования актуальные вопросы, достижения и инновации Сборник статей XXV Международной научно-практической конференции, Пенза МЦНС «Наука и просвещение». Россия, 10 июня 2022. - С.19-23. https://naukaip.ru/wp-content/uploads/2022/06/MK-1427-1.pdf
32. Khuzhayorov B., Fayziev B., Begmatov T. Suspension filtration model in a dual-zone porous medium with “charging” effect // Abstracts of the Uzbekistan-Malaysia international conference Computational models and technologies, September 16-17, 2022. Tashkent. 116-117 pp. https://cmt.nuu.uz/media/menu/ABSRACTBOOK-CMT2022_B.pdf
33. Fayziev B. Mathematical model of suspension filtration in porous media // International conference mathematical analysis and its applications in modern mathematical physics September 23-24, 2022; Samarkand, Uzbekistan. 154-155 pp.
34. Xo'jayorov B., Fayziyev B., Ermamatova Z. Gelmgols tenglamasi uchun chegaraviy teskari masalani yechish, № DGU 22158, O‘zbekiston Respublikasi Intellektual Mulk Agentligi, Toshkent, 2023.

35. Fayziev B., Salimova Sh. Mathematical modeling the process ofsuspension filtration in porous medium// “Matematika, mexanika va intellektual texnologiyalar” mavzusida yosh olimlar va tadqiqotchilarning respublika ilmiy konferensiyasi materiallari, Toshkent, UzMU, 28-29 mart 2023 yil
36. Kenjayev R.E., Fayziyev B.M. Koʻp fazali, koʻp komponentali suyuqlikning gʻovak muhitda sizishi jarayonining matematik modeli // “Zamonaviy axborot, kommunikatsiya texnologiyalari va at-ta’lim tatbiqi muammolari” mavzusidagi respublika ilmiy-amaliy anjumani ma’ruzalar toʻplami, Samarqand, 7-8 aprel 2023-yil, 259-261.
37. Fayziyev B., Tursunova Sh. “G’ovak muhitlarda ikki komponentali suspenziya sizishi jarayonining matematik modeli.” “Matematik modellashtirish va axborot texnologiyalarining dolzarb masalalari” xalqaro ilmiy-amaliy anjuman tezislar to’plami. Nukus. 2-3-may 2023.170-172b.
38. Shodmonov I., Fayziev B., Fayzieva M. A model of two-component suspension filtration in dual-zone porous medium// Matematik modellashtirish va axborot texnologiyalarining dolzarb masalalari” xalqaro ilmiy-amaliy anjuman tezislar to’plami. Nukus. 2-3-may 2023. 31 32b
39. Shodmonov I., Fayziev B. Fayzieva M. A mathematical model of two-component suspension in dual-zone porous medium // “II Republican Scientific and Practical Conference of Young Scientists” mathematics, mechanics and intellectual technologies, Nukus, 2-3 may 2023-yil, 60-62 b.
40. Tursunova Sh., Fayziev B. G`ovak muhitlarda ikki komponen tali suspenziya sizishi jarayonining “gibrid” matematik modeli // Fizika, matematika va mexanikaning dolzarb muammolari, Xalqaro ilmiy-amaliy anjumani materiallari, Buxoro, O’zbekiston, 24-25-may, 2023-yil. 66-68 b.
41. Kenjayev R, Babenko M, Fayziyev B. G‟ovak muhitda neft va suvning birgalikda sizishi jarayonining matematik modeli. // Fizika, matematika va mexanikaning dolzarb muammolari, Xalqaro ilmiy-amaliy anjumani materiallari, Buxoro, O’zbekiston, 24-25-may, 2023-yil. 46-49
42. Fayziev B., Begmatov T., Sagdullaev O. Mathematical model of solute transport in porous media with multistage deposition kinetics // Actual problems of applied mathematics and information technologies”-Al-Khwarizmi 2023. Samarkand, 46 b. https://staff.tiiame.uz/storage/users/953/articles/Cx4sBu23DbiPOtUHqqg0VfGuQ7nUOhqCrr860a2a.pdf
43. Fayziev B. Combined deep bed and cake filtration model // Actual problems of applied mathematics and information technologies”-Al-Khwarizmi 2023. Samarkand, 45b. https://staff.tiiame.uz/storage/users/953/articles/Cx4sBu23DbiPOtUHqqg0VfGuQ7nUOhqCrr860a2a.pdf
44. Sagʻdullayev O., Fayziyev B. Ikki zonali gʻovak muhitda “yemirilish” hodisasini hisobga olgan holda modda koʻchishi modeli masalaning qoʻyilishi // “Zamonaviy analiz va matematik fizika masalalari – ZAMFM – 2024”. 2024-yil 16-17-sentabr. Samarqand - 2024. 386-388 b.

45. Tursunova Sh., Fayziyev B. Go‘vak muhitda modda ko‘chishining gibrid matematik modeli // “Zamonaviy analiz va matematik fizika masalalari – ZAMFM – 2024”. 2024-yil 16-17-sentabr. Samarqand - 2024. 384-386 b.
46. B. Fayziev, O. Sag‘dullayev. Degrading Solute Transport in a Two-Zone Porous Medium with Multistage Adsorption Kinetics // International scientific conference, Actual problems of applied mathematics and information technologies - Al-Khwarizmi 2024, 22-23 October 2024, Tashkent, Uzbekistan. 44 – 45 page.
47. B. Fayziev, O.Sag‘dullayev, Sh. Salimova. Ko‘p bosqichli kinetika tenglamasi asosida g‘ovak muhitda degradatsiyalanadigan modda ko‘chishi modeli // “Avtomatlashtirish tizimlari va yashil energetika muammolari: ishlab chiqarishda, fan va ta’limda”, Qarshi - 2024 yil, 18-19 oktabr. 30 – 33 b.
48. B. Fayziev, O.Sag‘dullayev, N. Turopova. G‘ovak muhitda “yuklanish” hodisasini hisobga olgan holda modda ko‘chishining matematik modeli // “Ta’lim jarayoniga raqamli texnologiyalar va zamonaviy usullarni joriy etish muammolari”, Kattaqo’rg’on 18-19 aprel, 2024 y. 871 -875 b.
49. B. Fayziev, O. Sag‘dullayev, B. Baxtiyorov. Ikki zonali g‘ovak muhitda “eskirish” hodisasini hisobga olgan holda modda ko‘chishi modeli // “Ta’lim jarayoniga raqamli texnologiyalar va zamonaviy usullarni joriy etish muammolari”, Kattaqo‘rg‘on 18-19 aprel, 2024 y. 866 -870 b.
50. Xujayorov B., Fayziev B. M., Sag‘dullayev O. Q. G’ovak muhitlarda modda ko’chishi jarayonini monitoring qilish uchun dasturiy vosita // O‘zbekiston Respublikasi Adliya vazirligining elektron hisoblash mashinalari uchun yaratilgan dasturning rasmiy ro‘yxatdan 02.10.2024 yil o‘tkazilganligi to‘g‘risida guvohnoma. № DGU 42618
51. B. Khuzhayorov, B.Fayziev, O.Sagdullaev. Modeling the transport of a degrading solute in a two-zone porous medium with decay // Zamonaviy matematikaning dolzarb muammolari va uning tatbiqlari, 10-11 sentabr, 2025, Toshkent, 322 -323 – b.
52. Fayziev B. M., Haydarov O. RadDetectPro – radioktiv ifloslanish manbasi intensivligini aniqlash dasturi // O‘zbekiston Respublikasi Adliya vazirligining elektron hisoblash mashinalari uchun yaratilgan dasturning rasmiy ro‘yxatdan 04.08.2025 yil o‘tkazilganligi to‘g‘risida guvohnoma. № DGU 54152
53. Fayziev B. M., Tursunova Sh., Haydarov O. Suspenziya sizishi gibrid modelining sonli tahlili uchun dasturiy ta'minot // O‘zbekiston Respublikasi Adliya vazirligining elektron hisoblash mashinalari uchun yaratilgan dasturning rasmiy ro‘yxatdan 07.07.2025 yil o‘tkazilganligi to‘g‘risida guvohnoma. № DGU 53739

Avtoreferat “Sharof Rashidov nomidagi Samarqand davlat universiteti tahririy-nashriyot bo‘limi” tahririyatida tahrirdan o‘tkazildi va o‘zbek, ingliz, rus (rezyume) tillardagi matnlari mosligi tekshirildi (05.08.2026).

2026-yil 5-avgustda bosishga ruxsat etildi:
Ofset bosma qog‘ozi. Qog‘oz bichimi 60x84 $^{1}/_{16}$.
“Times” garniturasi. Ofset bosma usuli.
Shartli b.t. 4,0. Adadi 00 nusxa. Buyurtma 000.

“Sardor poligraf” OK bosmaxonasida chop etildi.
Manzil: Samarqand viloyati, Samarqand tumani, Xishrav MFY.

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